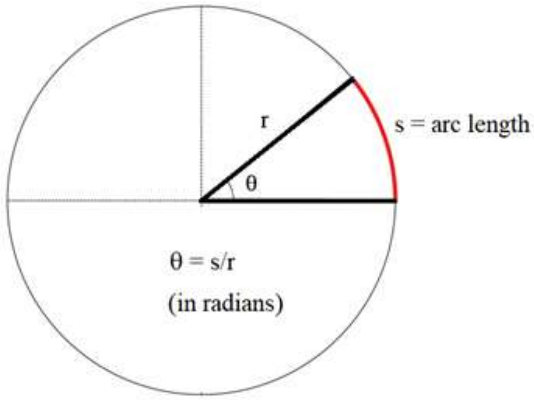
Physics 25 Angular Magnification Dr. Alward

Review: Radians



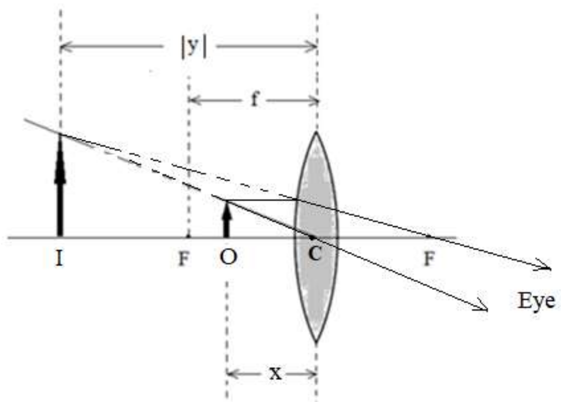
θ is the angle subtended by the arc length s from the center of the circle. If θ is less than nine degrees (0.160 rad), $\theta \cong \tan\theta$.

The table below shows that the smaller the angle, the more accurately does the tangent of the angle approximate the angle:

θ (radians)	$\tan\theta$
0.5200	0.5726
0.3500	0.3650
0.1700	0.1717
0.0900	0.0902
0.0400	0.0400

Review: The Lens Equations (See Chapter 26)

The diagram below shows a virtual, upright, and taller image of an object that is placed between the focal point and a convex lens. This is the configuration that will concern us in this chapter.



Lens Equations:

$1/x + 1/y = 1/f$
$M = -y/x$
$H_I = MH_o$

The image distance y is negative if the image is not on the eye-side.

In the figure at the left, y is negative, so the distance of the image from the lens is |y|.

The Magnifying Glass

A jeweler places a loupe* in his eye to grade a diamond in the face of a watch. The image appears a distance |y| from the lens, as shown in the figure at the right.

*A "loupe" is a small magnifying glass used by jewelers and watchmakers.

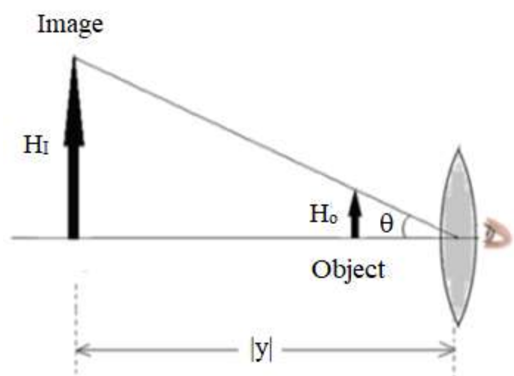


Figure above is not to scale.

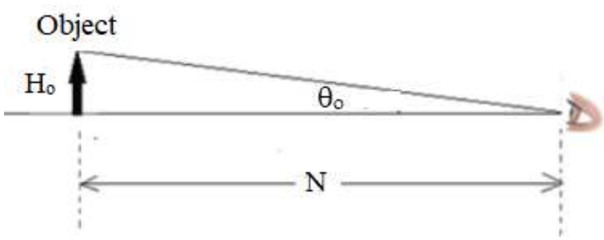
The "angular size" of the image is defined to be the angle θ subtended by the image from the center of the lens. The object height is much smaller than the image height, so the angle θ can be approximated by its tangent:

$\theta \cong \tan\theta$
$\quad = H_I / |y|$

This angular size will be compared below to the greatest angular size achievable with a lens without eye strain, and without the aid of a lens.

The Reference Angular Size

The reference angular size is defined to be the angular size an object has when the object is at the jeweler's near point, i.e., a distance N from the eyes.



The reference angular size is symbolized θ_o. Assuming the object height H_o is small compared to the near point distance N, the tangent of the angle is a good approximation to the angle.

$$\theta_o \cong \tan\theta_o$$
$$= H_o/N$$

Angular Magnification of a Magnifying Glass

In this section we compare the angular size of the image created with the convex lens to the reference angular size.

Recall:

$\theta = H_I / |y|$
$\theta_o = H_o / N$

Define angular magnification:

$$\begin{aligned} M_A &= \theta/\theta_o \\ &= (H_I/|y|)/(H_o/N) \\ &= (M)H_o/(-y)/(H_o/N) \\ &= (-y/x)\, H_o/(-y)/(H_o/N) \\ &= N(1/x) \\ &= N(1/f - 1/y) \\ &= N(1/f + 1/|y|) \end{aligned}$$

Summary:

There are two different forms of the angular magnification equation:

1. $M_A = N/x$
2. $M_A = N(1/f + 1/|y|)$

<u>Example</u>:

A jeweler whose near point is 40 cm is using a loupe attached to his eye to examine a diamond. The focal length of the loupe's lens is 5 cm.

The jeweler's eyes can comfortably focus on any image at or beyond 40 cm. After positioning the diamond (varying x) to obtain an acceptable magnification, the image is 185 cm from his eye:

$y = -185$ cm
$|y| = 185$ cm

(a) What is the angular magnification of the image?

$$\begin{aligned} M_A &= N(1/f + 1/|y|) \\ &= 40\,(1/5 + 1/185) \\ &= 8.22 \end{aligned}$$

(b) How far is the diamond from the lens?

$M_A = N/x$
$8.22 = 40/x$
$x = 4.87$ cm

Example A:

A jeweler whose near point is 50 cm is using a loupe attached to his eye to examine a diamond. The focal length of the loupe's lens is 6 cm.

(a) What would be the angular magnification if the diamond were 5 cm from the lens?

$M_A = N/x$
$= 50/5$
$= 10$

(b) Where is the image?

$1/5 + 1/y = 1/6$
$y = -30$ cm

(c) Could the image be seen without eye strain?

No. It cannot be clearly seen, because the image is nearer to the eye than the near point.

Example B:

Jeweler's near point: $N = 40$ cm
Loupe's focal length: $f = 5.00$ cm

What is the largest angular magnification possible without the jeweler suffering eyestrain?

$M_A = N\,(1/f + 1/|y|)$

The equation above makes it clear that, for a given f, the smaller $|y|$ is, the greater is M_A.

The smallest $|y|$ can be without causing eye strain is when the image is at the near point:

$|y| = 40$ cm $\quad y = -40.0$ cm

$M_A = 40.0\,(1/5.00 + 1/40.0)$
$= 9.00$

<u>Example</u>:

A jeweler's near point is 60 cm from the lens. He uses a loupe whose focal length is 20 cm to grade a diamond. The diamond's image has an angular magnification of 7.60.

(a) How far from the lens is the diamond?

$M_A = N(1/x)$
$7.60 = 60\ (1/x)$
$x = 7.89$ cm

(b) Can the jeweler clearly see the image?

$1/y = 1/f - 1/x$
$= 1/20 - 1/7.89$
$y = -13.03$ cm
$|y| = 13.03$ cm

No. The image is too close to the eye, whose near point is 60 cm from the eye.

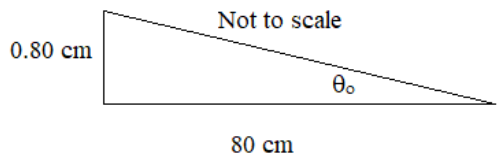
Example:

An object whose height is 0.80 cm is slowly moved from far away toward the unaided eye; when the approaching object reaches a point closer than 80 cm from the eye, eyestrain is produced as the eye struggles to see the object clearly. In other words, the near point for this observer is 80 cm from the lens: N = 80 cm.

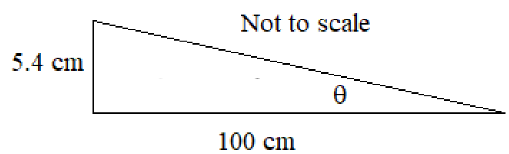
When a magnifying glass is then placed next to the eye, and the same object is placed 100 cm from the lens, the image height is 5.4 cm.

(a) What is the angular magnification of the image?

Calculate the reference angular size:
N = 80 cm (the near point distance)
$\theta_o \cong \tan \theta_o$
$= 0.80/80$
$= 0.01$ rad



Calculate the image angular size:
$\theta \cong \tan \theta$
$= 5.4/100$
$= 0.054$ rad



Finally, calculate the angular magnification:

$\theta/\theta_o = 0.054/0.01$
$= 5.4$

(b) The focal length of the lens is 20 cm. How far from the lens is the image?

$M_A = N\,(1/f - 1/y)$
$5.4 = 80(1/20 - 1/y)$
$y = -57.14$ cm
$|y| = 57.14$ cm

(c) Can the image be viewed without eye strain?

No. The image is closer to the eye than the 80-cm near point.