Physics 25 Chapters 31-32 Nuclear Physics

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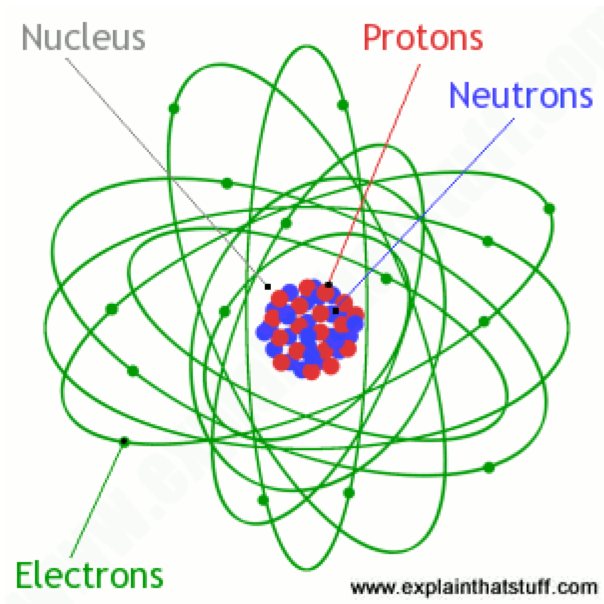
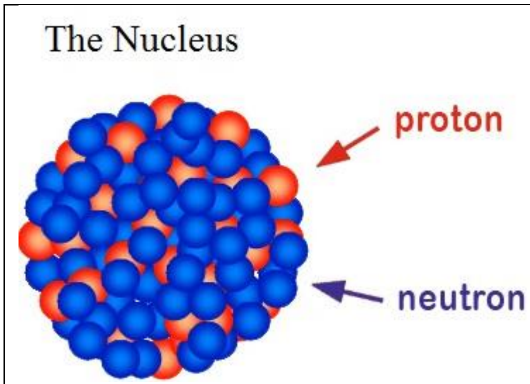
Video Lecture 1: Nuclear Decay
Video Lecture 2: Radio-carbon Dating
Video Lecture 3: Dating a Fossil
Video Lecture 4: Decay Problem
Video Lecture 5: Chain Reactions
Video Lecture 6: Radiation Dose Problem



	Z = Atomic Number = Number of Protons N = Number of Neutrons A = Atomic Mass (Weight) = Z + N = Number of nucleons The residents of nuclei--protons and neutrons--are called “nucleons.”

Examples of Nuclei Symbols

The table below shows three of the more common ways of symbolizing nuclei. Subscripts in the table below are the atomic numbers, i.e., the number of protons. Superscripts are the atomic masses--the number of nucleons.

Element	Z	N	A	Symbol	Symbol	Symbol
carbon	6	6	12	$_6C^{12}$	carbon-12	C-12
nitrogen	7	8	15	$_7N^{15}$	nitrogen-15	N-15
uranium	92	146	238	$_{92}U^{238}$	uranium-238	U-238
helium	2	2	4	$_2He^4$	helium-4	He-4

Isotopes

Every element occurs in a variety of different forms, called "isotopes." For example, consider carbon. There are several isotopes of carbon all of them chemically and electrically indistinguishable from each other. All of the isotopes of carbon have the same atomic number, i.e., the same number of protons, but different numbers of neutrons. The table below lists four of the fifteen isotopes of carbon.

Element	Z	N	A	Symbol	Symbol	Symbol	Percent Abundance
carbon	6	6	12	$_6C^{12}$	carbon-12	C-12	98.93
carbon	6	7	13	$_6C^{13}$	carbon-13	C-13	1.07
carbon	6	8	14	$_6C^{14}$	carbon-14	C-14	---
carbon	6	9	15	$_6C^{15}$	carbon-15	C-15	---

As the table above indicates, most of the carbon in the universe is carbon-12; about one percent of it is carbon-13. The abundances of the other carbon isotopes are negligibly small; carbon-14, for example, makes up only one out of a trillion carbon nuclei.

Elementary Particles and Their Symbols

Subscripts in the table below are the charges (as a multiple of e).
Superscripts are the number of nucleons.

Name	Symbol	Description
alpha particle	${}_{2}\alpha^{4}$	Two protons and two neutrons
electron	${}_{-1}e^{o}$	elementary particle: no constituents
beta particle	${}_{-1}\beta^{o}$	Electron emitted in beta decay
positron	${}_{1}\beta^{o}$	Same as the beta particle, except positive.
neutron	${}_{0}n^{1}$	Two down quarks and one up quark.
proton	${}_{1}p^{1}$	Two up quarks and one down quark.
gamma photon	${}_{o}\gamma^{o}$	A “bundle” of electromagnetic energy

Students should memorize these symbols.

Radioactive Decay of Unstable Nuclei

If the ratio of neutrons to protons in a nucleus is too large, or too small, the nucleus is said to be “unstable.”

Unstable nuclei become more stable by changing their neutron-proton ratio, which is accomplished by radiating away (emitting) one or more of the elementary particles listed in the table above. This process is called “radiation” and the nucleus is said to be “radioactive” and undergoing “decay.”

Nuclei are also sometimes unstable because they contain more energy than they can tolerate, perhaps by virtue of having absorbed gamma radiation; they can become more stable by emitting “gamma-ray” photons. Nuclei also can become more stable by emitting one or more of the particles listed above.

Balancing Radioactive Decay Equations

Balancing Rules

The sum of subscripts on each side of the time arrow is the same, and the sum of the superscripts is likewise the same.

Examples

Proton emission	$_{27}Co^{53} \rightarrow {}_{26}Fe^{52} + {}_{1}p^{1}$
Neutron emission	$_{4}Be^{13} \rightarrow {}_{4}Be^{12} + {}_{0}n^{1}$
Beta particle emission	$_{6}C^{14} \rightarrow {}_{7}N^{14} + {}_{-1}\beta^{0}$
Positron emission	$_{12}Mg^{23} \rightarrow {}_{11}Na^{23} + {}_{1}\beta^{0}$
Alpha emission	$_{86}Rn^{222} \rightarrow {}_{84}Po^{218} + {}_{2}\alpha^{4}$
Gamma ray emission	$_{53}I^{131} \rightarrow {}_{54}Xe^{131} + {}_{-1}\beta^{0} + {}_{o}\gamma^{o}$

Note that the subscripts and superscripts rules hold true for the six decays described in the table above.

Beta Particle Emission (Beta Decay)

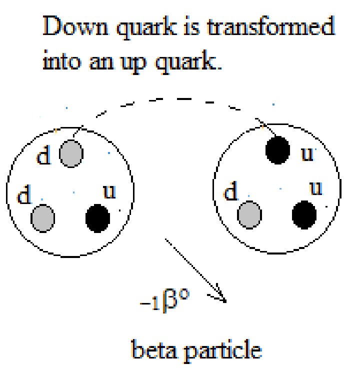
Beta particles were so-named before it was realized that the particle being emitted in beta decay actually was an electron. But, wait. How can an electron be emitted from a nucleus if the only residents of a nucleus are protons and neutrons?

The answer is below.

Beta decay occurs when one of the neutrons in the nucleus is spontaneously transformed into a proton.

${}_0n^1 \rightarrow {}_1p^1 + {}_{-1}\beta^o$

A closer look is provided below. One of the two down quarks in the neutron spontaneously is transformed into an up quark with the simultaneously emission of an electron (a beta particle).



The event above is described in decay-equation form below:

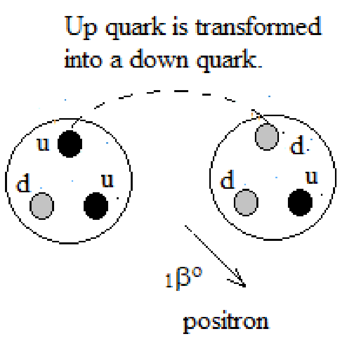
$${}_{-1/3}d^o \rightarrow {}_{2/3}u^o + {}_{-1}\beta^o$$

Positron Emission

Positron emission occurs when one of the protons in the nucleus is transformed into a neutron:

$_1p^1 \longrightarrow {}_0n^1 + {}_1\beta^o$

The proton above becomes a neutron when one of its up quarks is spontaneously transformed into a down quark, with the simultaneous emission of a positron.

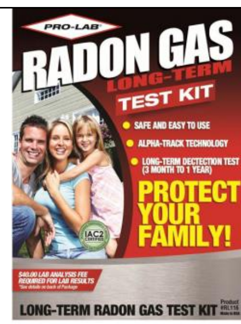


$_{2/3}u^o \longrightarrow {}_{-1/3}d^o + {}_1\beta^o$

Uranium Decay Chain and Radon Gas Poisoning

Radon, a dangerous alpha-emitting gas, is created in Earth's crust according to the decay scheme shown below.

$$\begin{array}{ccccc} {}_{92}U^{238} & \xrightarrow{\alpha} & {}_{90}Th^{234} & \xrightarrow{{}_{-1}\beta^{o}} & {}_{91}Pa^{234} \\ & & & & \downarrow {}_{-1}\beta^{o} \\ {}_{88}Ra^{226} & \xleftarrow{\alpha} & {}_{90}Th^{230} & \xleftarrow{\alpha} & {}_{92}U^{234} \\ \downarrow \alpha & & & & \\ {}_{86}Rn^{222} & \xrightarrow{\alpha} & & & \end{array}$$

Radon gas rises upward from Earth's crust through foundation cracks and other openings of houses where it is breathed by the home's occupants. Radon bombards vital lung tissue with alpha particles, causing DNA mutations which often lead to lung cancer. Radon gas is the second-leading cause of lung cancer in the United States.	

Blocking Harmful Radiation

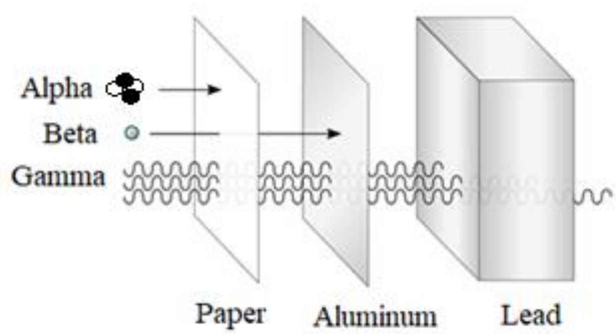
Radiation in the form of elementary particles, such as protons, neutron, beta particles (electrons), positrons, and gamma photons can be blocked or attenuated by placing a wall or barrier between the source and vulnerable living things.

The figure below shows three types of radiation that can be blocked by suitably thick or dense barriers.

1. Beta particles can be blocked by a couple of millimeters of aluminum.

2. Alpha particles have twice the charge of beta particles, but they are two-thousand times larger, so they don't penetrate barriers as far as do the smaller beta particles. Alpha particles can be blocked by clothing or even a thin sheet of paper.

3. Gamma photons have the greatest penetration power: To completely block the most energetic gamma radiation several inches of lead, or feet of concrete are necessary.



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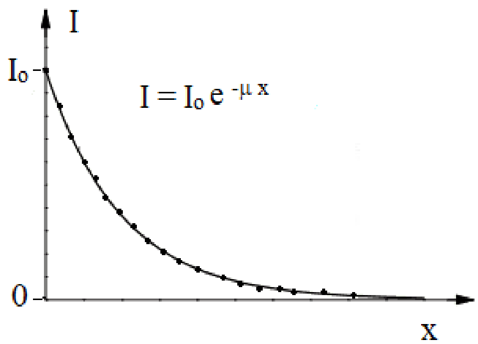
A calculation of the thickness of concrete required to "attenuate" gamma radiation intensity is given below.

Radiation Attenuation

Most substance are capable of absorbing radiation. The property of substances that relate to the efficiency with which they absorb the radiation is called the "absorption coefficient," which depends on the type of radiation. μ = Absorption Coefficient Units: cm^{-1}

Radiation Attenuation Equation

I = Radiation Intensity
Units: W/m^2



Example:

Radiation is incident on some substance. After penetrating it 2.30 cm, the intensity has been attenuated by 75%, i.e., only 25% remains. What is the absorption coefficient?

Recall that the logarithmic and the exponential functions are the inverses of each other:

$\ln(e^x) = x$
$e^{\ln(x)} = x$

Returning now to the problem:

$$I = I_o e^{-\mu x}$$
$$I/I_o = e^{-\mu x}$$
$$0.25 = e^{-\mu(2.30)}$$
$$\mu = \mathbf{0.603\ cm^{-1}}$$

Example:

A block of concrete faces a gamma source; the radiation intensity at the front face of the block is 0.30 W/m^2.

After penetrating 9.0 cm into the concrete, the intensity is reduced to 0.10 W/m^2.

What is the concrete's gamma radiation absorption coefficient?

$$I = I_o e^{-\mu x}$$
$$0.10 = 0.30 e^{-\mu (9.0 \text{ cm})}$$
$$1/3 = e^{-\mu (9.0 \text{ cm})}$$
$$\ln (1/3) = \ln [e^{-\mu (9.0 \text{ cm})}]$$
$$= -\mu (9.0 \text{ cm})$$
$$\mu = 0.122 \text{ cm}^{-1}$$

Half-Life

The "half-life" of a radioactive substance is the average amount of time it takes for half the sample to decay. Half-lives range from picoseconds (trillionths of seconds) to billions of years.

Half-Life Symbol: T

The half-lives of a few isotopes are shown in the table.

Isotope	Half-Life
Iodine-131	8 days
Radium-213	3 minutes
Radium-226	1600 years
Carbon-12	stable
Carbon-14	5730 years
Uranium-238	4.5 billion years
Radon-222	3.8 days
Radon-219	3.6 seconds

Example A:	Example B:
There are now 80 micrograms (μg) of radium-213 in a patient's thyroid gland. How many micrograms will there be 12 minutes from now? The half-life of Ra-213 is 3 minutes. 12 minutes = 4 half-lives 4 halvings of 80 μg: 80→ 40→ 20→ 10→ 5 μg	There are 15 picograms (pg) now of a certain isotope. 6.0 minutes ago, there were 120 pg. What is this isotope's half-life? (A pico-gram is a trillionth of a gram.) Answer: 120 → 60 → 30 → 15 Three halvings occur over a 6.0-minute time span, so the time required per halving is 2.0 minutes. T = 2.0 min

Example C:

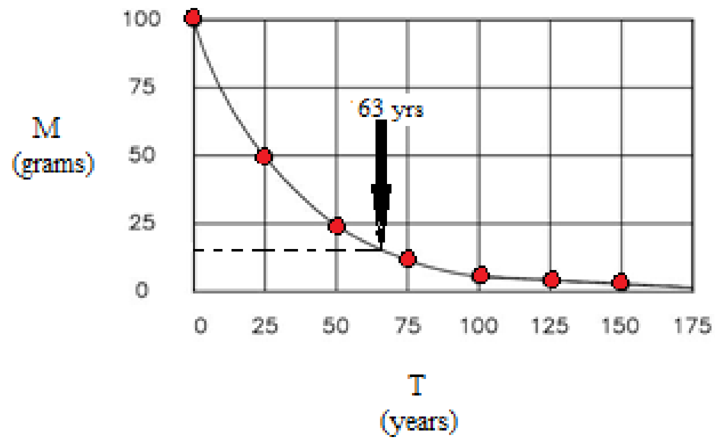
Strontium-90 is radioactive, with a half-life of about 25 years. Initially, there are 100 grams of Sr^{90}. How many grams would be left after 125 years (5 half-lives)?

After five halvings, only 3.12 grams would be left:

100→50→25→12.5→6.25→3.12

Exponential Decay

Data relating to the decay of strontium-90, whose half-life is 25 years, is shown in the table below. Use the data to construct the graph of the isotope's decay:



t (years)	M (grams)
0	100
25	50
50	25
75	12
100	12.5
125	6.25
150	3.12

<u>Example:</u>

How many grams M of the strontium-90 sample above will remain after 63 years?

63 is not an integer-multiple of strontium-90's 25-year half-life, so the solution cannot be obtained by making successive halvings, as was done in the previous examples. Instead, we estimate from the graph that M lies within the range (15 - 20) grams.

A method that allows one to obtain an *exact* answer is described next.

Exponential Decay Equation and the Decay Constant

The equation whose graph is the one shown above has the following form:

$M = M_o e^{-\lambda t}$ (Equation 1)

M_o = Initial Mass
M = Final Mass
λ = Decay Constant

Use the known value of the half-life of isotopes to calculate the decay constant.

Let T = Half-Life

When $t = T$, $M = \frac{1}{2} M_o$
Substitute into Equation 1:
$\frac{1}{2} M_o = M_o e^{-\lambda T}$

$\frac{1}{2} = e^{-\lambda T}$
Take the natural log of both sides:

$\ln(\frac{1}{2}) = \ln(e^{-\lambda T})$
$-0.693 = -\lambda T$

$\lambda = 0.693/T$

<u>Example</u>:

Use the radioactive decay equation to determine the number of grams of strontium that would be left after 63 years, assuming that there were 100 grams initially.

T = 25 years

$\lambda = 0.693/T$
$= 0.693/25$ years
$= 0.0277$ years^{-1}

$M = 100\, e^{-0.0277(63)}$
$= 17.5$ grams

This more accurate value matches well the estimated 15-20 grams estimated earlier from the graph.

Carbon-14

The carbon-14 atoms in the atmosphere combine with oxygen to form carbon dioxide, which plants absorb naturally and incorporate into plant fibers by photosynthesis. Animals eat plants and take in carbon-14 as well as carbon-12, and humans eat the plants *and* the animals, and thereby take in carbon-14 and carbon-12.

Carbon-12 doesn't decay, so its abundance in the *biosphere** doesn't change. Carbon-14 is unstable, and therefore decays. Beginning about 60,000 years ago the rates of production and decay of C-14 in the biosphere became equal, so its abundance in the biosphere is likewise constant. The ratio of carbon-12 to carbon-14 in living things has been the same for the last 60,000 years: about one trillion ($1.0 \times 10^{12)}$ to one.

**The biosphere is the portion of Earth, including atmosphere and oceans, rivers, lakes, and streams, where life exists.*

Carbon Refresh

Every day, our bodies take in new atoms from the air we breathe, the food we eat, and the liquids we drink. These atoms are incorporated into our cells. Carbon atoms in living things are "refreshed" about every six months, so the ratio of normal carbon (carbon-12) to carbon-14 in living things is the same as it is in the biosphere: one trillion to one.

Carbon-14 Dating

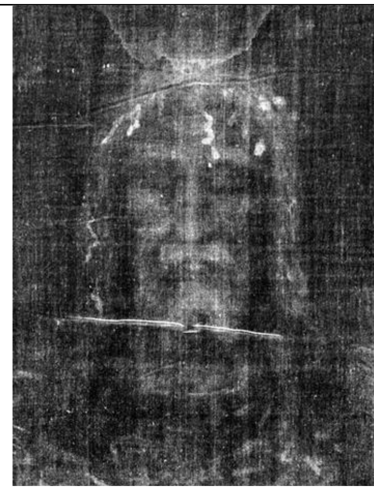
The ratio of carbon-12 to carbon-14 in *living* things is one trillion to one; when death occurs, carbon intake ceases because eating, breathing, and drinking ceases. The carbon-12 in the deceased object remains unchanged as time passes because carbon-12 doesn't decay, but the carbon-14 content halves every 5730 years. Therefore, the carbon-12/carbon 14 ratio *doubles* every 5730 years.

Example: The ratio of C-12 to C-14 in a fossil is four trillion to one. How old is the fossil?

Answer: A quadrupling of the ratio is two doublings, which is caused by two halvings of the carbon-14, so the animal stopped having its carbon refreshed (when the animal stopped breathing and eating) 11,460 years ago.

Radiocarbon dating of the Shroud of Turin proves that the fibers used to make the cloth came from flax plants that were uprooted (died) about six hundred years ago, which is about same time the shroud first made its appearance. Thus, the image on the shroud had to have been placed there as recently as about 600 years ago.

The Shroud currently resides in the Cathedral of Saint John the Baptist in Turin, in northern Italy. The Catholic Church, not disputing the science, makes no claim that the cloth is the burial cloth of *anyone* that lived about 2000 years ago, but nevertheless states that the cloth is "important as an icon of faith."



Carbon-14 Doubling Time Equation

Chemists measure the ratio R of a fossil's carbon isotopes C-14 and C-12, then use the equation below to determine the number of years ago when the animal died.

$R = 2^{t/T}(1.0 \times 10^{12})$, where T = 5730 years.

Example:

How many years after death will the ratio of carbon-12 to carbon-14 be 7.6×10^{13}?

Answer:

$7.6 \times 10^{13} = (2^{t/5730})(1.0 \times 10^{12})$
t = 35,800 years

Induced Radioactivity

"Induced" radioactivity occurs when nuclei collide with other particles, such as neutrons, protons, and alpha particles, or absorb photons, all of which often causes (induces) the nucleus to become more unstable, and thus radioactive.

A few examples are given below.

$_0n^1 + {_7N^{14}} \rightarrow {_7N^{15}} \rightarrow {_6C^{14}} + {_1p^1}$
$_2\alpha^4 + {_4Be^9} \rightarrow {_6C^{12}} + {_0n^1}$
$_0n^1 + {_{27}Co^{59}} \rightarrow {_{27}Co^{60}} \rightarrow {_{-1}\beta^o} + {_{28}Ni^{60}} + {_0\gamma^o}$

Chain Reaction in an Atomic Bomb

The three events in the sequence below represent the "trigger" that initiates the chain of "fissioning" uranium-235 that occurs when an atomic bomb (A-Bomb) is detonated.

First, radioactive polonium-210 emits alpha particles. Next, the alpha particles collide with beryllium-9 nuclei, which become radioactive and emit neutrons. Finally, the neutrons collide with uranium-235 which initiate a "chain-reaction" of exponentially-multiplying "fissions" of uranium-235.

$_{84}Po^{210} \rightarrow {_{82}Pb^{206}} + {_2\alpha^4}$

$_2\alpha^4 + {_4Be^9} \rightarrow {_6C^{12}} + {_0n^1}$

$_0n^1 + {_{92}U^{235}} \rightarrow {_{36}Kr^{91}} + {_{56}Ba^{143}} + {_0n^1} + {_0n^1}$ + energy

Energy Release in Uranium-235 Fission

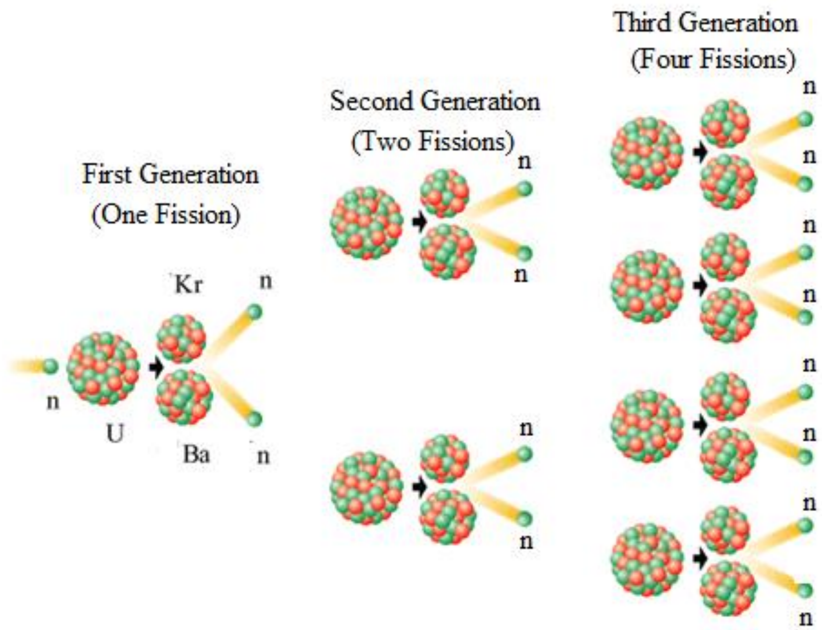
The mass of the products in the fission reaction above is less than the mass of the reactants by 2.2×10^{-28} kg. The disappeared mass changes into a quantity Q of heat and electromagnetic energy, shown below:

$$
\begin{aligned}
Q &= (\Delta m)\, c^2 \\
&= (-2.2 \times 10^{-28}\ \text{kg})(3.0 \times 10^{8})^2 \\
&= -1.98 \times 10^{-11}\ \text{J}
\end{aligned}
$$

This is about three million times the heat released when the combustion of one molecule of octane gasoline occurs. The explosive power of vapor from one gallon (≅ 2.5 kg) of gasoline, mixed with oxygen and ignited, is capable of flattening a three-bedroom house. Imagine three *million* such house-flattenings from 2.5 kg of uranium-235 undergoing fission. (A 3.5-kg uranium sphere has a diameter of about two inches.)

Chain Reaction in Fissioning U-235

In the diagram below, the first "generation" of fissions occurs when a neutron from decaying beryllium collides with a uranium-235 nucleus. That single fission then causes two fissions in the second generation, then those two fissions cause four fissions in the third generation, and so on.



The *total* number of fissions that will have occurred after N generations is symbolized as S_N, and is given below. Note: the first "generation" (N = 1) occurs when the beryllium neutron (the neutron at the far in the figure above) collides with the first uranium nucleus; the remaining generations are born when neutrons emitted from uranium nuclei go on to cause fissions.

$$S_N = 2^N - 1$$

N	S_N
1	1
2	3
3	7
4	15
10	1023
20	1,048,576

The chain reaction described above for U-235 assumes the *ideal* case in which each fission causes two more fissions. In this ideal case we may say the "neutron reproduction rate" is 2.0. Let's symbolize this rate as R:

$R = 2.0$

In general, for arbitrary R,

$S_N = (R^N – 1)/(R – 1)$

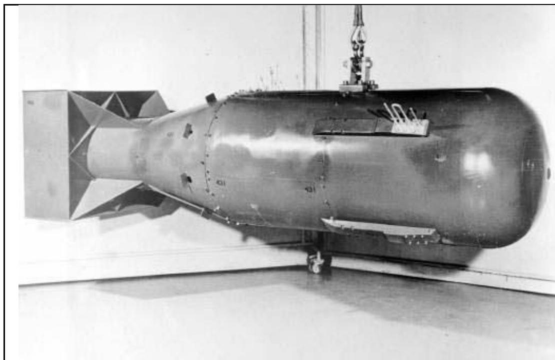
Each generation of fissioning urantium-235 takes about one *attosecond* to be born.
(One attosecond = 1.0×10^{-18} s.)

Example:

Suppose R = 1.7. After how many attoseconds would two million fissions have occurred?

$2.0 \times 10^6 = (1.7^N – 1)/(1.7 – 1)$
$N = 26.67$

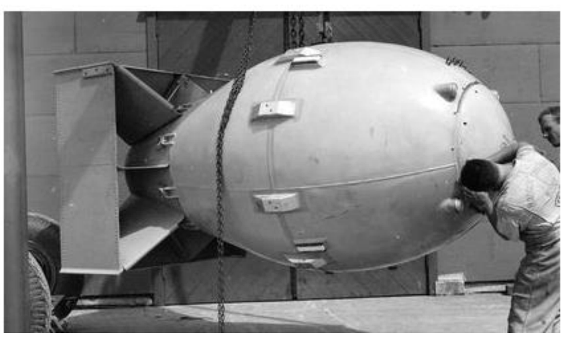
Little Boy and Fat Man



Little Boy

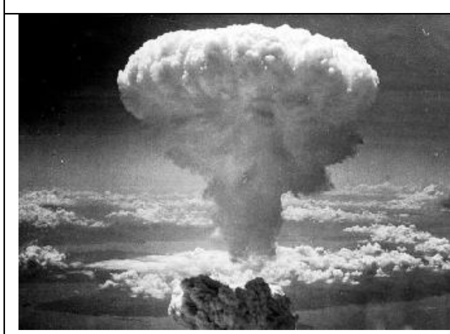
The atomic bomb named “Little Boy” was dropped on Hiroshima, Japan, August 6, 1945. One neutron fired into a clump of uranium the size of a shoebox 1900 feet above the ground killed 60,000 persons instantly and caused the deaths of 110,000 more from radiation poisoning in the subsequent years.

A second atomic bomb, a larger one, called “Fat Man,” was dropped on Nagasaki, Japan, a few days later; its design was different, and plutonium--not uranium--was the fissionable fuel used in that bomb.



Fat Man

A B-29 Bomber, named “Enola Gay,” dropped the Hiroshima bomb.



Radiation Dose

D = “Dose”
Units: Joules/Kilogram

1.0 J/kg = 100 Rads

Example A: A 60 kg person absorbed 300 joules of gamma radiation. What dose (in rads) did she receive?

300 J/ 60 kg = 5.0 J/kg
= 500 rads

Geiger counters measures radiation intensity.

Example B:	Example C:
A 70-kg person absorbs 280 J of radiation. What dose does she receive?	A person receives a dose of 800 rads. How many joules per kilogram is this?
D = 280 J / 70 kg = 4 J/kg = 4 (100 rad) = 400 rad	800 rad = 800 (0.01 J/kg) = 8.0 J/kg

Biological Effects of Absorbed Radiation

Dose (rads)	J/kg	Health Consequence
100	1.0	Vomiting within a week, hair falls out, then full recovery
150	1.5	Death within a week without care; with care, eventual recovery.
320	3.2	50% die within 60 days, if minimal care
480	4.8	50% die within 60 days, assuming supportive medical care
1100	11.0	50% die within 60 days, assuming intensive medical care

Example:

The output power of a spherically-symmetric radioactive gamma source is 5.0 W. A 60-kg person whose effective uncovered surface area facing the source is 0.70 m^2. She is 2.0 meters from the source.

(a) What is the intensity of the gamma radiation at the person's location?

$I = P/ 4\pi r^2$
$= (5\text{ W}) / [4\pi (2.0\text{ m})^2]$
$= 0.0995\text{ W/m}^2$

(b) What quantity (in joules) of radiation energy does she absorb per second?

$[0.0995\text{ (J/s)/m}^2\text{]}\ 0.70\text{ m}^2 = 0.0696\text{ J/s}$

(c) What quantity Q of radiation energy will she have absorbed in three hours ?

3(3600 = 10,800 s

Q = (0.0696 J/s) (10,800 s)
= 752 J

(d) What is this person's dose in J/kg?

752 J/60 kg = 12.53 J/kg

(e) What is her dose in rads?

12.53 J/kg = 12.53 (100 rad)
= 1253 rad