Physics 25 Chapter 30

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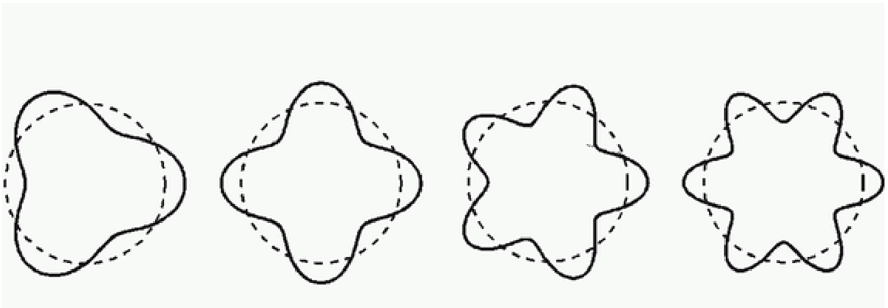
Video Lecture 1: Quantization of Orbital Radii
Video Lecture 2: Quantization of Energy
Video Lecture 3: Excitation and De-Excitation
Video Lecture 4: Problem 6 Chapter 30

Electron Energies in the Hydrogen Atom

Electrons cannot exist in a circular orbit around the hydrogen nucleus unless an integer number of electron deBroglie wavelengths fit exactly around the circumference, otherwise the matter wave will interfere with itself.



n = 3 n = 4 n = 5 n = 6

Orbital Radii in Hydrogen are Quantized

"Quantized" means only certain radii are allowed.

Constructive Interference Condition

$2\pi r = n\lambda$
$= n\,(h/mv)$
$v = nh/2m\pi r$ (Equation 1)

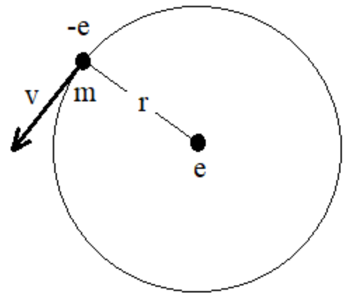
Newton's 2nd Law
$ke^2/r^2 = mv^2/r$ (Equation 2)

Insert the expression for v from Equation 1 into Equation 2, and solve for r:

$r = n^2\,(h^2/4\pi^2 ke^2 m)$
$= n^2\,(5.33 \times 10^{-11}$ meters$)$
$= n^2\,(0.53$ Å$)$

Orbital radii are "quantized" based on the "quantum number", n:

$r_n = n^2$ (0.53 Å), n = 1, 2, 3, …



$m = 9.11 \times 10^{-31}$ kg
$k = 9 \times 10^{9}$ N-m^2/C^2
$e = 1.6 \times 10^{-19}$ C
$h = 6.63 \times 10^{-34}$ J-s
1.0 Å = 1.0×10^{-10} m

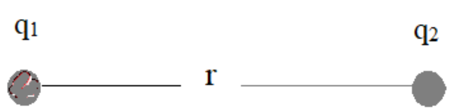
Orbital radii that are not one of the quantized ones are forbidden. Electrons attempting to orbit with a radius that's not one of the ones allowed "quantum mechanically" would destructively interfere with themselves, so such orbits are never occupied.

For example, orbital radii such as r = 2.13 Å, or r = 8.50 Å, cannot exist.

n	$r_n = 0.53\,n^2$ (Å)
1	0.53
2	2.12
3	4.77
4	8.48

In what follows, we will explore the quantization of the *energies* of the electron in hydrogen.

Hydrogen Electron Potential Energy U



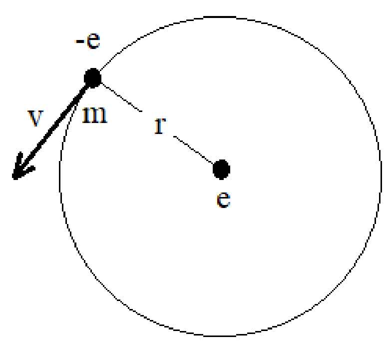
Without proof, we state here that the electric potential energy of a system of two charged objects a distance r apart is:

$U = kq_1q_2 /r$

For the hydrogen atom, $Q_1 = e$ and $Q_2 = -e$:

$U = -ke^2/r$

Hydrogen Electron Kinetic Energy



$ma = F$

$mv^2/r = ke^2/r^2$

$mv^2 = ke^2/r$

$½\ mv^2 = ½\ ke^2/r$

$K = ½\ ke^2/r$

Example:

Use the equation above to calculate the kinetic energy of an electron in the orbit of hydrogen whose radius is 4.77 Å.

$r = 4.77 \times 10^{-10}$ m
$K = ke^2/2r$
$= (9 \times 10^9)(1.6 \times 10^{-19})^2/(2 \times 4.77 \times 10^{-10})$
$= 2.41 \times 10^{-19}$ J
$= (2.41 \times 10^{-19}$ J$) / 1.6 \times 10^{-19}$ J/eV)
$= 1.51$ eV

This is the same result we obtained by a different method in Chapter 29.

Total Energy Quantization

Obtain an expression for the total energy E of the hydrogen electron in terms of the orbital radius, r.

Recall from the expression for kinetic energy K from above:

$K = ke^2/2r$

$U = - ke^2/r$

$E = K + U$
$= ke^2/2r - ke^2/r$
$= ke^2/2r - 2ke^2/2r$
$= - ½\, ke^2/r$
$= -½\, (9 \times 10^9)\, (1.6 \times 10^{-19})^2 /[n^2\, (5.3 \times 10^{-11})\,]$
$= (-2.17 \times 10^{-18} /n^2)$ J

Convert to electron-volt energy units:

$E = [(-2.17 \times 10^{-18}$ J $)/n^2\,]/ 1.6 \times 10^{-19}$ J/eV
$= (-13.6$ e/V$)/n^2$

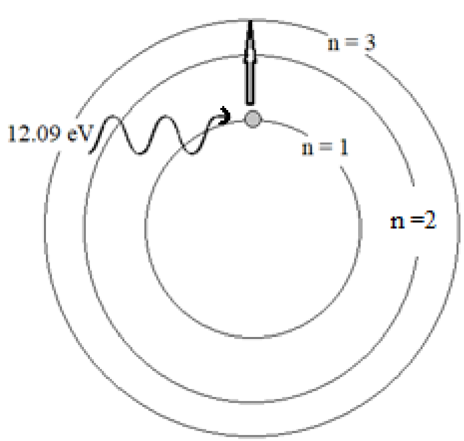
$\mathbf{E_n = -13.6}$ **eV/$\mathbf{n^2}$**, n = 1, 2, 3, ….

Table of Quantized Energies

n	E_n $-13.6/n^2$ (eV)	Name	Other Name
1	-13.6	Ground state	1st quantum level
2	-3.34	1st excited state	2nd quantum level
3	-1.51	2nd excited state	3rd quantum level
4	-0.85	3rd excited state	4th quantum level
5	-0.54	4th excited state	5th quantum level

Electrons occupying "lower-lying" quantum states, such as n =1, or n = 2, can be "excited" up into "higher-lying" states by absorbing the energy of an incident photon. Only certain photon energies will cause excitation. The new electron energy MUST be one of the allowed energies. For example, the electron below in the ground state can absorb a 12.09 eV photon because the new electron energy is one of the allowed ones: -13.6 eV + 12.09 eV = -1.51 eV. Other photon energies would not excite the electron. For example, if the photon energy were 12.19 eV, the new electron energy would be -13.6 eV + 12.19 eV = -1.41 eV; this energy is not one of the allowed ones, so the photon energy would be rejected.

Electron Excitation



Radii are not drawn to scale.

Example A:

What is the wavelength of light that will excite an electron in the n = 1 state to the n = 3 state?

Conservation of energy tells us the following:

Electron's Final Energy = Initial Energy + Photon Energy Absorbed

$$E_3 = E_1 + 1243/\lambda$$

$$-13.6/3^2 = -13.6/1^2 + 1243/\lambda$$

$\lambda = 102.81$ nm

Light whose wavelength is not this exact value would not be absorbed by the electron, otherwise E_3 would be a value that's not allowed.

Example B:

What frequency of light must be shined on a gas of hydrogen atoms in order to cause 2-5 transitions?

Photon Energy + $E_2 = E_5$

$$1243/\lambda - 13.6/2^2 = -13.6/5^2$$

$\lambda = 434.6$ nm

$= 4.346 \times 10^{-7}$ m

$f = c/\lambda$

$= (3.0 \times 10^8) / (4.346 \times 10^{-7})$

$= 6.90 \times 10^{14}$ Hz

Ionization

<u>Example</u>:

(a) What is the diameter d of hydrogen in its normal state--its ground state?

d = 2 (0.53 Å)
= 1.06 Å

(b) How much light energy would an electron in its ground state have to absorb to excite it into the n = 100 quantum state?

$-13.6/100^2 = -13.6/1^2 +$ Photon Energy
Photon Energy = 13.6 eV

(c) How far from the proton is the electron in the n = 100 state?

$r_{100} = 100^2$ (0.53 Å)
= 5300 Å
= 5000 d

Thus, the excited electron is very, very far from the hydrogen proton--five thousand times the diameter of hydrogen in its ground state. Effectively, the electron has been completely removed from hydrogen atom, leaving behind the hydrogen ion. We say the hydrogen has been "ionized."

13.6 eV is said to be "the ionization energy" of hydrogen.

De-Excitation

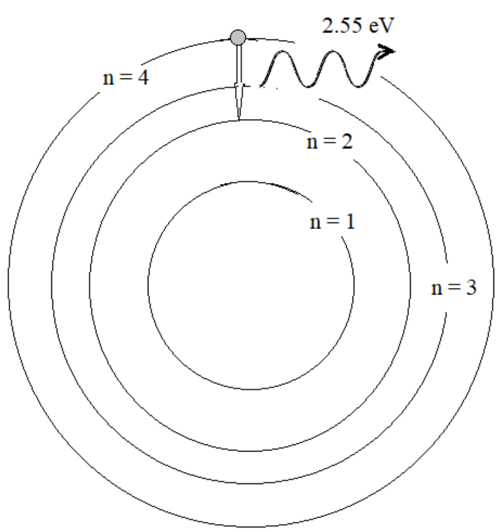
Electrons in an excited state “de-excite” to a lower-lying energy state by emitting a photon that -will lower the electron’s energy to a value that exactly matches the energy of the lower-lying energy. This process is sometimes also called “relaxation.”

The figure below shows the relaxation of an electron in the fourth quantum state down to the second quantum state. The final energy of the electron equals its initial energy, minus the energy it lost when it emitted a photon:

$$-13.6/2^2 = -13.6/4^2 - \text{Photon Energy}$$

Photon Energy = 2.55 eV

Electron De-Excitation



Radii are not drawn to scale.

Example:

A gas of hydrogen atoms is heated. Consider just those hydrogen atoms that are in the fourth quantum (n = 4) level. What possible wavelengths of light will be emitted during the time the atoms in the fourth quantum level have relaxed down to the ground state following all possible intermediate transitions?

Final Electron Energy = Initial Electron Energy - Photon Energy

$$E = E_o - 1243/\lambda$$

$$\lambda = 1243 / (E - E_o)$$

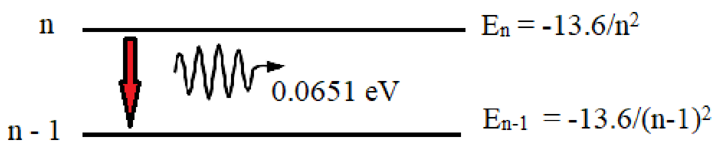
Transition	E_o Initial Energy (eV)	E Final Energy (eV)	$E - E_o$ Photon Energy (eV)	$1243/(E - E_o)$ λ (nm)
4-3	-0.85	-1.51	0.66	1883
4-2	-0.85	-3.40	2.55	487
4-1	-0.85	-13.60	12.75	97
3-2	-1.51	-3.40	1.89	658
3-1	-1.51	-13.60	12.09	103
2-1	-3.40	-13.60	10.20	122

Example:

A relaxing (de-exciting) electron in hydrogen relaxes down to its nearest neighbor orbit and emits a photon whose energy is 0.0651 eV. What transition occurred?

Let n be the unknown quantum number of the initial excited state.

The electron relaxes down to the n-1 state:



Initial Electron Energy - Photon Energy = Final Electron Energy

$$-13.6/n^2 - 0.0651 = -13.6/(n-1)^2$$
$$n = 8$$

The transition that occurred was n = 8 to n = 7.