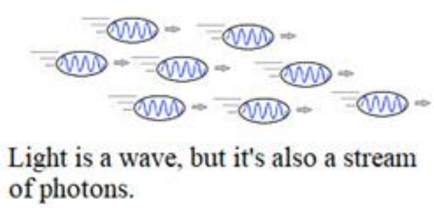
Physics 25 Chapter 29 Dr. Alward

Video Lecture 1: Duality of Light, Photons
Video Lecture 2: Photon Energy, 1243 Equation
Video Lecture 3: Photoemission
Video Lecture 4: Photon Momentum, de Broglie Wavelength
Video Lecture 5: Matter Waves Explain Electron Energy

Photons, Photoemission, and Matter Waves

Max Planck



Albert Einstein

The work of Max Planck and Albert Einstein led to the suggestion that light sometimes acts like a wave, and other times like a stream of mass-less, particle-like, point-sized "packets," or "bundles" of electromagnetic energy, called "photons," traveling at the speed of light, c.

When light is manifesting itself as a wave, it has the properties of a wave: frequency, and wavelength.

$$f = \text{frequency}$$
$$\lambda = c/f$$
$$\text{or } f = c/\lambda$$

When light manifests itself as like a stream of particles (photons), the photons each travel at the speed of light ($c = 3.0 \times 10^8$ m/s). Each photon in the stream has energy and a momentum:

<u>Photon Energy</u>: Photons have an energy E that is proportional to the frequency of the light manifested when the light is traveling as a wave. The constant of proportionality is called "Planck's Constant", h, where $h = 6.63 \times 10^{-34}$ J-s.

$$E = hf$$

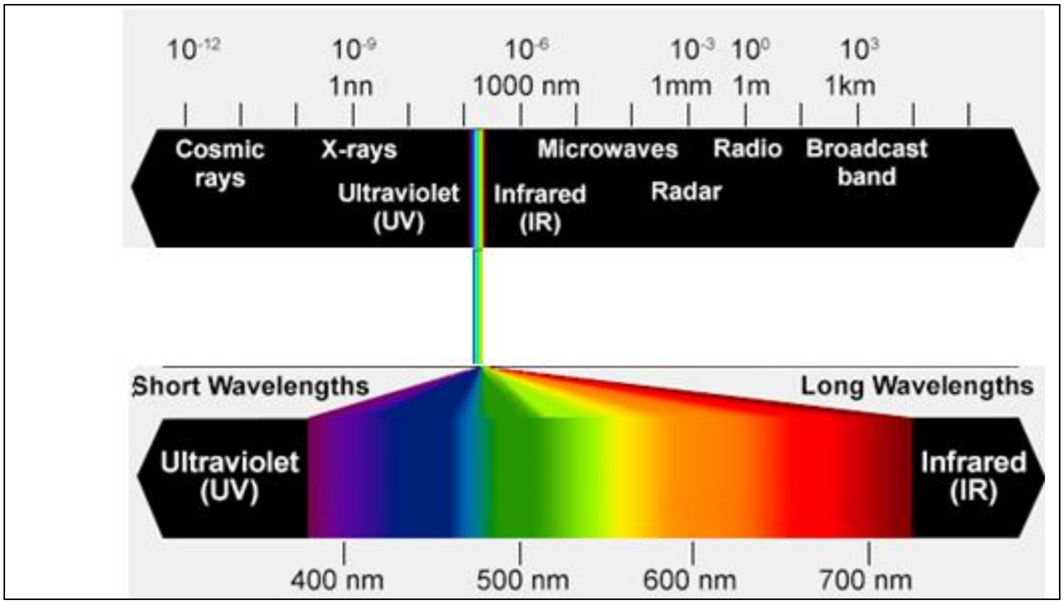
Photon Momentum

When light is manifesting as particle-like entity, and it is incident on surface, it interacts with the surface just as if it were a particle carrying momentum, p. Without proof, we state here that the momentum of a photon in a beam of light whose energy is E, is

$$p = E/c$$

More discussion of photon momentum is given on a following page.

Below is the electromagnetic spectrum discussed in Chapter 24:



Example:

What is the energy (in eV) of photons in 720 nm red light?

$E = hf$

$= h (c/ \lambda)$

$= (6.63 \times 10^{-34}) (3.0 \times 10^{8}) / (720 \times 10^{-9})$

$= 2.76 \times 10^{-19}$ J

$= 2.76 \times 10^{-19}$ J $/1.6 \times 10^{-19}$ J/eV

$= 1.73$ eV

Example:

What is the wavelength (in nm) in a beam light consisting of a stream of 3.0 eV photons?

$E = 3.0 \text{ eV } (1.6 \times 10^{-19} \text{ J/eV})$
$\quad = 4.8 \times 10^{-19} \text{ J}$

$f = E/h$
$\quad = 4.8 \times 10^{-19}/6.63 \times 10^{-34}$
$\quad = 7.24 \times 10^{14} \text{ Hz}$
$\lambda = c/f$
$\quad = 3.0 \times 10^{8} /7.24 \times 10^{14}$
$\quad = 4.14 \times 10^{-7} \text{ m}$
$\quad = 414 \times 10^{-9}$
$\quad = 414 \text{ nm}$

The 1243 Equation

There is a quick way of determining photon energies and wavelengths:

$E = hf$
$\quad = hc/\lambda$

$hc = (6.63 \times 10^{-34} \text{ J-s})(3.0 \times 10^{8} \text{ m/s})$
$\quad = 1.99 \times 10^{-25} \text{ J-m}$
$\quad = 1.99 \times 10^{-25} \text{ J-m}/1.6 \times 10^{-19} \text{ J/eV}$
$\quad = 1.243 \times 10^{-6} \text{ eV-m}$
$\quad = (1.243 \times 10^{-6} \text{ eV-m}) / (1.0 \times 10^{-9} \text{ m/nm})$
$\quad = 1243 \text{ eV-nm}$

$E = hc/\lambda$
$\quad = (1243 / \lambda) \text{ eV-nm}$

In order to obtain E in eV, λ must be in nm, which will cancel the nm units. The two problems on the previous page are solved at the right using the 1243 rule.

Example A:

Red light of wavelength $\lambda = 720$ nm consists of a stream of photons. What is the energy of each photon (in eV)?

$E = 1243/720$
$\quad = 1.73 \text{ eV}$

Example B:

What is the wavelength (in nm) of the light consisting of a stream of 3.0 eV photons?

$\lambda = 1243/3.0$
$\quad = 414 \text{ nm}$

Example:

A laser pen emitting green light of wavelength λ = 530 nm is operating at an output power of 40.0 milli-watts. How many photons are emitted each second?

E = 1243/530
   = 2.35 eV

$(40.0 \times 10^{-3}$ J/s) /$(1.6 \times 10^{-19}$ J/eV) = 2.50×10^{17} eV/s

$(2.50 \times 10^{17}$ eV/s) / (2.35 eV/photon) = 1.06×10^{17} photons/s

The Photoelectric Effect

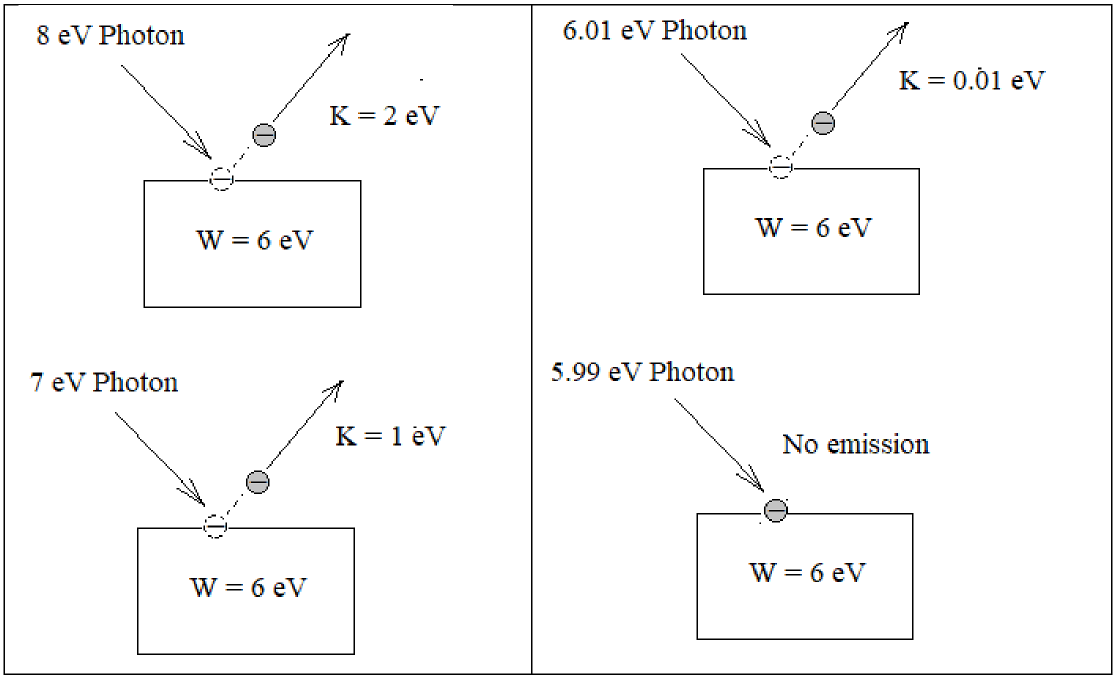
The least photon energy E that will cause electrons at a metal's surface to escape from the surface is called "the work function" W. If the electron escapes, its kinetic energy is given by the equation below

K = E - W

The examples below show photoemission from a metallic surface whose work function is 6 eV. Note that if the photon has an energy less than the work function, the electron does not escape the surface. The kinetic energy a photo-emitted electron has equals is whatever energy is left over after spending the required work-function energy to escape the surface:

K = E - W

The four examples below illustrate photoemission.



All of the examples above assume the electron was already at the surface of the metal; electrons initially a distance below the surface may also capture a photon and perhaps then photo-emitted. Such electrons might not have enough energy left over after traveling to the surface to have any energy left over to over-come the work function to escape.

<u>Example</u>:

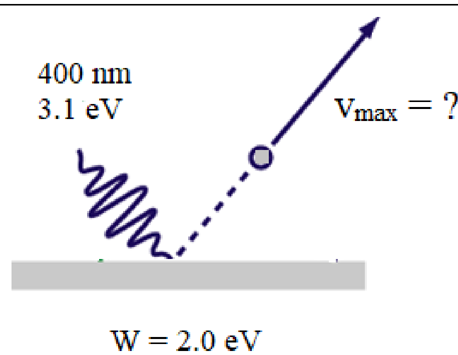
The work function of potassium is 2.0 eV. What is the speed of emitted electrons, initially at the surface of potassium, when 400 nm light is shined on the surface?

The mass of electrons is 9.11×10^{-31} kg.

$E = (1243)/400$
$\quad = 3.1$ eV

$K = E - W$
$\quad = 3.1 - 2.0$
$\quad = 1.1$ eV
$\quad = (1.1 \text{ eV}) (1.6 \times 10^{-19} \text{ J/eV})$
$\quad = 1.76 \times 10^{-19}$ J

$\frac{1}{2} (9.11 \times 10^{-31}) v^2 = 1.76 \times 10^{-19}$
$v = 6.22 \times 10^5$



Example:

The least frequency of light that will photo-emit electrons from the surface of a metal is 3.0×10^{14} Hz. What is the kinetic energy of electrons, initially at the surface, that are photoemitted when the surface is illuminated in light whose frequency is 4.0×10^{14} Hz?

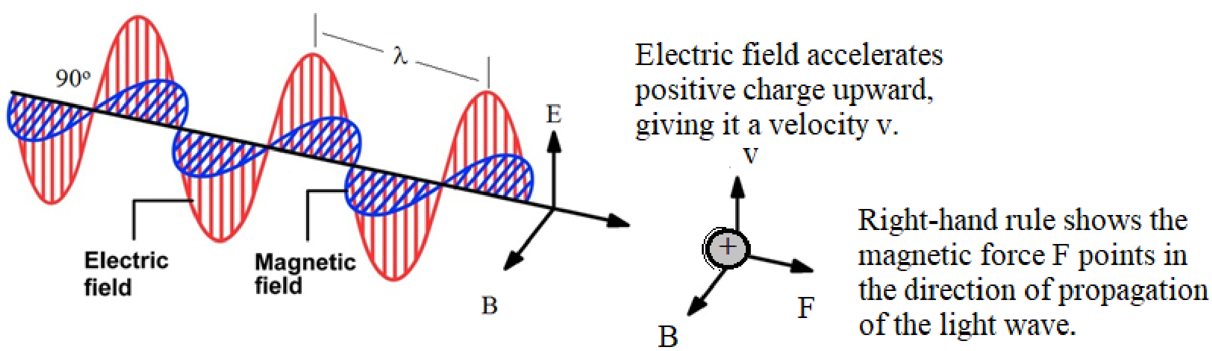
$W = hf - K$

The least frequency f will be the one for which the photon energy hf is as small as it can be while still allowing the electron to escape the surface with barely detectable kinetic energy--virtually zero K:

$W = hf - 0$
$= (6.63 \times 10^{-34})(3.0 \times 10^{14}) - 0$
$= 1.99 \times 10^{-19}$ J

$K = hf - W$
$= (6.63 \times 10^{-34})(4.0 \times 10^{14}) - 1.99 \times 10^{-19}$
$= 6.62 \times 10^{-20}$ J

Photon Momentum



The light wave above, encountering a proton, exerts a force on it according to the magnetic force right-hand rule:

The direction of the force is obtained with the right-hand rule to be in the same direction of propagation of the wave.

Thus, an electromagnetic wave exerts a force on the proton just as if the light wave had momentum--which it does, as noted below:

Recall that we earlier noted that a photon of energy E has momentum:

$p = E/c$

Replacing E with hc/λ

$p = (hc/\lambda) /c$
$\quad = h/\lambda$

The equation above shows that wave-like entity (a wavelength) can have a particle-like attribute (a momentum, p).

Is the reverse possible? de Broglie twisted the equation around to obtain,

$$\lambda = h/p$$

Could it be shown that moving objects behave like waves? Does a rolling marble, for example, have a wavelength?

de Broglie Waves

(also called "Matter Waves")

In 1924, graduate student Louis de Broglie speculated that if a wave-like entity (light) can have a matter-like property (momentum), as we saw above, then maybe the reverse is also true: Matter has a wave-like property--a wavelength.

$\lambda = h/p$

p = Momentum of the object
λ = Wavelength of the "matter-wave"

(λ is called "the de Broglie wavelength," or, alternatively, "the matter-wavelength.")

<u>Example</u>:

What is the matter-wavelength (in angstroms) of an electron moving at 2.0×10^6 m/s?

Electron mass: $m = 9.11 \times 10^{-31}$ kg

1.0 angstrom (Å) = 1.0×10^{-10} m

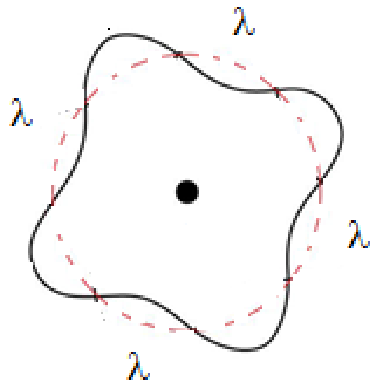
$p = mv$
$= (9.11 \times 10^{-31}\text{ kg})(2.0 \times 10^{6}\text{ m/s})$
$= 1.82 \times 10^{-24}$ kg-m/s

$\lambda = h/p$
$= 6.63 \times 10^{-34}/ 1.82 \times 10^{-24}$
$= 3.64 \times 10^{-10}$ m
$= 3.64$ Å
Note: one Angstrom (A) = 1.0×10^{-10} m.

To put this length in perspective, the separation between atoms in solids is roughly 1.0 - 3.0 Å.

Constructive Interference of Electron Matter-Waves

The electron orbiting the hydrogen nucleus exhibits "constructive interference" of its matter wave with itself. This is illustrated below for the case of four matter-wavelengths arranged neatly end-to-end, fitting neatly around the circumference of an orbit.



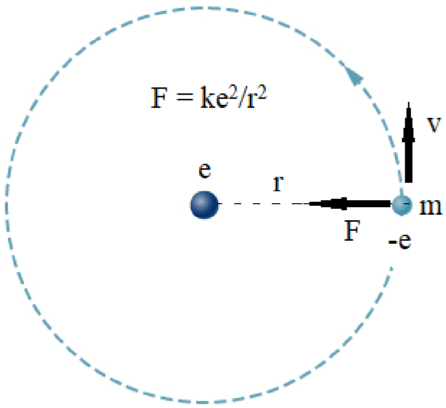
As illustrated above, exactly four wavelengths around the circumference of the circle:

$$4\lambda = 2\pi r$$

The general condition for constructive interference for electrons traveling in a circular orbit of radius r is shown below:

$$n\lambda = 2\pi r, \text{ where } n = 1, 2, 3, \ldots$$

Example:



Calculate the de Broglie wavelength of the electron in a hydrogen atom orbiting in a circular path of radius r = 2.12×10^{-10} m.

$\lambda = h/p$
$= h/(mv)$
Need to get v before we can use the values of h and m to solve the above equation for λ.

$F = ma$
$ke^2/r^2 = mv^2/r$
Solve for v:
$k = 9 \times 10^{9}$ N-m^2/C^2
$e = 1.6 \times 10^{-19}$ C
$m = 9.11 \times 10^{-31}$ kg

$v = 1.092 \times 10^{6}$ m/s

$p = mv$
$= 9.11 \times 10^{-31} \times 1.092 \times 10^{6}$
$= 9.95 \times 10^{-25}$

$\lambda = h/p$
$= (6.63 \times 10^{-34}) / (9.95 \times 10^{-25})$
$= 6.66 \times 10^{-10}$ m

Constructive and Destructive Interference

For the orbiting hydrogen electron discussed on the previous page, we explore below the issue of constructive and destructive *self*-interference of the electron matter wave:

Recall the radius of the orbit is 2.12×10^{-10} m.
Circumference = $2\pi r$
$= 2\pi (2.12 \times 10^{-10})$
$= 1.33 \times 10^{-9}$ m

How many matter-wavelengths (de Broglie wavelengths) fit around the 1.33×10^{-9} m circumference above?

Recall the de Broglie wavelength: $\lambda = 6.66 \times 10^{-10}$ m found earlier:

Answer:
$1.33 \times 10^{-9} / 6.66 \times 10^{-10} = 2.0$

Two de Broglie wavelengths fit neatly around the circular path, similar to the standing waves on a string stretched between two points, allowing the wave to constructively interfere with itself, creating a standing wave. A whole number of the hydrogen atom electron's de Broglie wavelength always fit neatly around the orbital circumference.

Kinetic Energy and Momentum

Going forward, it will be useful to have an expression for the kinetic energy K of an object in terms of its momentum p:

$K = ½ mv^2$
$= (mv)^2/(2m)$
$= p^2/2m$

Example:

What is the kinetic energy (in eV) of an electron in a circular orbit of radius 4.77 Å, given that three of the electron's matter wavelengths fit around the circumference?

Recall: 1.0 Å = 1.0×10^{-10} m
$m = 9.11 \times 10^{-31}$ kg

$n\lambda = 2\pi r$
$3\lambda = 2\pi (4.77 \times 10^{-10})$
$\lambda = 9.99 \times 10^{-10}$ m

$p = h/\lambda$
$= 6.63 \times 10^{-34} / 9.99 \times 10^{-10}$
$= 6.64 \times 10^{-25}$ kg-m/s

$K = p^2/2m$ (This is the equation derived earlier.)
$= (6.64 \times 10^{-25})^2 / (2 \times 9.11 \times 10^{-31})$
$= 2.42 \times 10^{-19}$ J
$= 2.42 \times 10^{-19}$ J $/ 1.6 \times 10^{-19}$ J/eV
$= 1.51$ eV