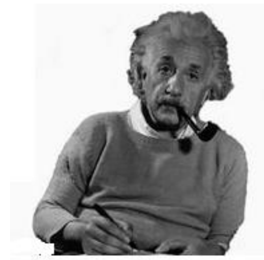
Physics 25 Chapter 28 Special Relativity

Dr. Joseph F. Alward



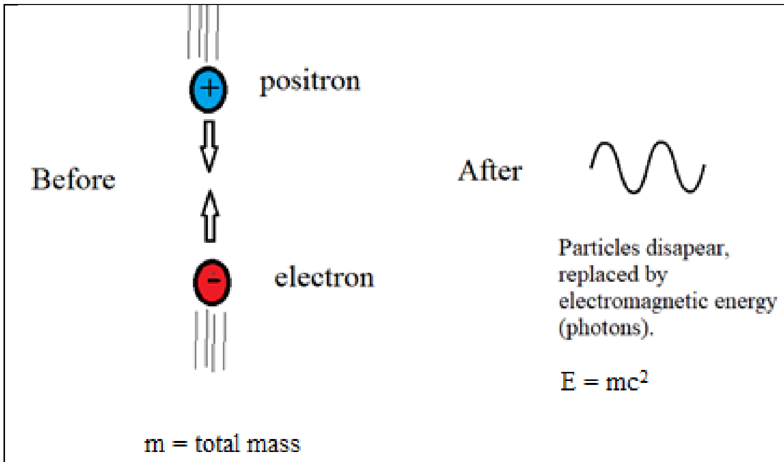
According to Albert Einstein's *Special Theory of Relativity*, mass and energy are two different forms of the same thing, called "mass-energy." Mass can be created out of energy, and energy out of mass. The energies encountered in this chapter typically are less than a trillionth of a joule; it is more convenient to use the unit of energy described below: MeV

Alternative Energy Units

1.0 electron-volt (eV) = 1.6×10^{-19} J
1.0 million eV (MeV) = 1.0×10^{6} eV
= 1.6×10^{-13} J

One example mass being converted to energy of is seen in the process called *electron-positron annihilation*. A positron is the antiparticle of an electron. It has all the properties of an electron except for the polarity of the electrical charge, which is positive. If an electron and a positron collide, each particle disappears and in their place appears an amount of electromagnetic energy given by the "Einstein Equation,"
$E = mc^2$.

Electron-Positron Annihilation



The sum of the two masses is m = 1.82 x 10^{-30} kg; the energy equivalent of this amount of mass is

$E = 1.82 \times 10^{-30} (3.0 \times 10^{8})^2$
$= 1.64 \times 10^{-13}$ J
$= 1.64 \times 10^{-13}$ J $/1.6 \times 10^{-13}$ J/MeV
$= 1.02$ MeV

Total Energy

According to the special theory, the total energy of an object moving with speed v is:

$E = mc^2 / (1 - \beta^2)^{1/2}$ where $\beta = v/c$ and $c = 3.00 \times 10^8$ m/ s

<u>Example</u>:

What is the total energy (in MeV) of a proton whose speed is four-tenths the speed of light?

$$\beta = 0.40$$
$$m = 1.67 \times 10^{-27} \text{ kg}$$

$$\begin{aligned} E &= mc^2 / (1 - \beta^2)^{1/2} \\ &= (1.67 \times 10^{-27})(3 \times 10^8)^2 / (1 - 0.40^2)^{1/2} \\ &= 1.64 \times 10^{-10} \text{ J} \end{aligned}$$

$(1.64 \times 10^{-10}$ J$)/ (1.6 \times 10^{-13}$ J/MeV$) = 1025$ MeV

Rest-Mass Energy

The total energy an object has when it is *at rest* relative to an observer, i.e., when $\beta = 0$, is called the “rest-mass energy.”

$$\begin{aligned} E &= mc^2 / (1 - 0^2)^{1/2} \\ &= mc^2 \end{aligned}$$

<u>Example</u>:

What is the rest-mass energy of protons?

Proton Mass: $m = 1.67 \times 10^{-27}$ kg

$$\begin{aligned} mc^2 &= (1.67 \times 10^{-27})(3.0 \times 10^8)^2 \\ &= 1.503 \times 10^{-10} \text{ J} / 1.6 \times 10^{-13} \text{ J/MeV} \\ &= 939 \text{ MeV} \end{aligned}$$

Relativistic vs Classical Kinetic Energy

Subtract the rest-mass energy mc^2 from the total energy E, and what's left is the *motional* (kinetic) part of the energy, i.e., the kinetic energy, K:

$$K = E - mc^2$$

$$K = mc^2 /(1- \beta^2)^{1/2} - mc^2$$

The kinetic energy we learned about in a previous semester, ½ mv^2, is called the "classical" kinetic energy, while this new K is called the "relativistic" kinetic energy. The relativistic kinetic energy is the correct kinetic energy, while the classical kinetic energy is only approximate.

At "non-relativistic" speeds (v less than one-tenth the speed of light), the relativistic kinetic energy is virtually the same as the classical kinetic energy, so either equation may be used without meaningful error. In that case, it's easier to use the classical equation, $K = ½ mv^2$.

Work-Kinetic Energy Theorem

Recall from Physics 23 the relationship between the total work done on an object and the resulting change in that object's kinetic energy.

Example:

How much work (in MeV) must be done to change the speed of a proton from $\beta_1 = 0.20$ to $\beta_2 = 0.60$?

$K_2 = 939 / (1 – 0.60^2)^{1/2}$
$= 1173.75$ MeV

$K_1 = 939 / (1 – 0.20^2)^{1/2}$
$= 958.36$ MeV

$W = K_2 – K_1$
$= 215.39$ MeV

Proton Moving at non-Relativistic Speed

Example:

Calculate and compare the classical and relativistic kinetic energies (in joules) of a proton (mc^2 = 939 MeV) traveling at the non-relativistic speed, v = 600 m/s

Classical:

$K = ½\ mv^2$
$= ½\ (1.67 \times 10^{-27})\ (600)^2$
$= 3.006 \times 10^{-22}$ J

Relativistic:

$\beta^2 = (600 / 3 \times 10^8)^2$
$= 4 \times 10^{-12}$

$K = mc^2 / (1- \beta^2)^{1/2} - mc^2$
$= 939 / (1 – 4 \times 10^{-12})^{1/2} - 939$
$= (4.6950 \times 10^{-4}$ MeV$)\ (1.6 \times 10^{-13}$ J/MeV$)$
$= 3.008 \times 10^{-22}$ J

Even at speeds as large as 600 m/s, the classical kinetic energy value is only about 0.007 % less than the relativistic value, so one is justified in using the classical kinetic energy equation, $K = ½\ mv^2$.

At much higher speeds, the classical kinetic energy is wildly inaccurate, as the following example illustrates.

Proton Moving at Relativistic Speed

At higher speeds, the classical kinetic energy is quite inaccurate, so the relativistic equation is the one that should be used. The following example illustrates how great can be the difference between the correct K—the relativistic one, and the inaccurate K, the classical one.

Obtain the kinetic energies in MeV.

Example:

Calculate and compare the classical and relativistic kinetic energies (in joules) of a proton (mc^2 = 939 MeV) traveling at the relativistic speed, $v = 1.2 \times 10^8$ m/s ($\beta = 0.40$).

Classical Kinetic Energy:

$$
\begin{aligned}
K &= \tfrac{1}{2} mv^2 \\
&= \tfrac{1}{2} (1.67 \times 10^{-27}) (1.2 \times 10^8)^2 \\
&= 1.20 \times 10^{-11} \text{ J} / 1.6 \times 10^{-13} \text{ J/MeV} \\
&= 75.15 \text{ MeV}
\end{aligned}
$$

Relativistic:

$$
\begin{aligned}
K &= mc^2 / (1-\beta^2)^{1/2} - mc^2 \\
&= 939 / (1 - 0.40^2)^{1/2} - 939 \\
&= 85.53 \text{ MeV}
\end{aligned}
$$

The classical kinetic energy is about 14% less than the correct kinetic energy.

Events

In what follows we will be discussing "events." For example, an event might be a runner beginning and completing a run, or a candle lighted and eventually burning out, or the falling of a cone breaking loose from a pine tree and landing on the ground, or the a space-traveler beginning a trip at one solar-system, and ending at a second one.

According to Einstein, different observers moving at different speeds, observing an event, disagree about how much time it took for the event to occur and both are correct.

Proper Time vs Dilated Time

T_o = Proper time

The "proper" time is the amount of time it takes for an event to occur according to an observer who is <u>at rest</u> relative to the location of the event.

T = Dilated time

The "dilated" time is the amount of time it takes for the event to occur according to an observer <u>moving</u> relative to the location of the event.

Both observers are correct; one's reality depends on one's speed relative to events. Such realities are true for each observer.

$T = T_o / (1 - \beta^2)^{1/2}$

Note: The denominator is less than 1.0, so the dilated time is greater than the proper time.

Note: "Proper" is somewhat of a misnomer. The proper time is not a more accurate, or better time measurement than the dilated time. Each is correct for the respective observers.

Example:

An observer holding an ice cube measures the time it took to melt to be 8.0 minutes. She is at rest relative to the location (her hand) of the event (the melting), so she measures the proper time:

T_o = 8.0 minutes

A second observer looking at the melting cube in the first observer's hand while moving at $\beta = 0.40$ says it took more than 8.0 minutes melt. She measures the dilated time.

$$T = T_o / (1 - \beta^2)^{1/2}$$
$$= 8.00 / (1 - 0.40^2)^{1/2}$$
$$= 8.73 \text{ minutes}$$

What is the correct time?
Answer: Both are correct.

Example:

A runner completes a journey while running 2.00 minutes, according to her. Does she measure the dilated time, or the proper time?

The runner is located where the event--the running-- is occurring, just like the person who held the ice cube in the previous example was located where the event--the melting--was occurring, so the runner measures the proper time.

T_o = 2.00 minutes

Suppose a passenger in a spaceship flying by the event at 0.80 c observes the running. How long will he say it took the runner to complete her run?

The passenger is moving relative to the location of the event (the runner's location), so he measures the dilated time:

$$T = 2.00 /(1 - 0.80^2)^{1/2}$$
$$= 3.33 \text{ minutes}$$

Both times are correct (for the respective observer.)

Length Contraction

Proper Length L_o

L_o = Length measured by an observer *at rest* relative to the object

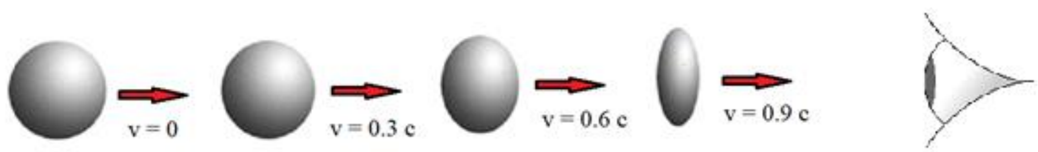
Contracted Length L

L = Length measured by an observer *moving* relative to the object
= "the contracted length"

$$L = L_o\,(1 - \beta^2)^{1/2}$$

Both lengths are correct for their respective observers. There are as many realities in nature as there are observers.

Contraction occurs in the dimension that is along the direction of motion of the moving observer. Below we show an object moving to the right, toward an observer. The narrowing of the object's width occurs along the object's direction of travel and depends on the object's speed.



Note: the word "proper" does not imply that the length measured by the stationary observer is the "real" length, and all other measurement are somehow flawed or "improper" because of errors by the observer, faulty equipment, or illusion. The length measured by the moving observer is just as real for that observer as it is for the observer who is at rest relative to the object.

Example:

A observer at rest relative to a bridge measures it to be 400 m. This is the proper length: $L_o = 400$ m

A traveler in a rocketship is traveling at 0.70 c relative to the bridge.`

(a) What does the traveler say is the length of the bridge?

The traveler is moving relative to the bridge, so he measures the contracted length:

$$L = L_o (1 - \beta^2)^{1/2}$$
$$= 400 (1 - 0.70^2)^{1/2}$$
$$= 286 \text{ m}$$

(b) How much time does the traveler say it took him to travel past the bridge?

Answer: He is located where the event (the traveling) is occurring, so he measures the proper time, T_o.

$$v = 0.70 (3.0 \times 10^8 \text{ m/s})$$
$$= 2.1 \times 10^8 \text{ m/s}$$

Time = Distance / Speed

$$T_o = 286 \text{ m} / 2.1 \times 10^8 \text{ m/s}$$
$$= 1.36 \times 10^{-6} \text{ s}$$

(c) How long does the stationary observer say the rocketship took to travel past the bridge?

$$T = T_o / (1 - \beta^2)^{1/2}$$
$$= 1.36 \times 10^{-6} / (1 - 0.70^2)^{1/2}$$
$$= 1.90 \times 10^{-6} \text{ s}$$

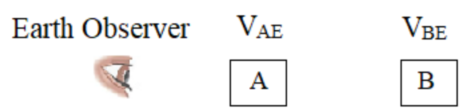
Each observer measures the correct time, for them.

Classical Relative Velocities

In classical physics, relative velocities are calculated as illustrated in the example below.

Let the velocity of an Object A relative to an Earth observer be V_{AE} and the velocity of Object B relative to the Earth observer be $V_{BE.}$

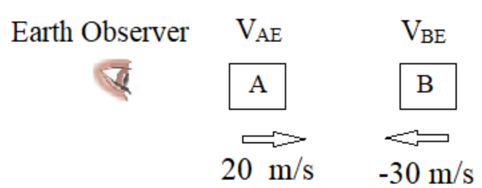
Velocities of objects moving toward Earth are negative, while objects moving away from Earth have positive velocities.



The velocity of Object A relative to Object B is

$V_{AB} = V_{AE} - V_{BE}$

<u>Example</u>: V_{AE} = 20 m/s and V_{BE} = -30 m/s



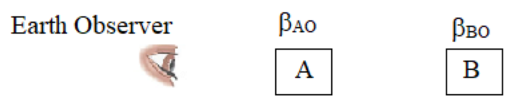
$$\begin{aligned} V_{AB} &= V_{AE} - V_{BE} \\ &= 20 - (-30) \\ &= 50 \text{ m/s} \end{aligned}$$

Observer B sees A moving toward it at a speed of 50 m/s.

Relative Velocities (Relativistic)

At very great velocities--greater than about one-hundredth the speed of light, velocities are said to be “relativistic.” Relative velocity calculations involving extremely high-speed objects must be done using the relativistic equations shown below; classical physics equations are not accurate.

The equation that is valid for relativistic as well as classical velocities is shown below for Objects A and B moving relative to an observer on Earth.



$$\beta_{AB} = (\beta_{AE} - \beta_{BE}) / (1 - \beta_{AE}\,\beta_{BE})$$

Example:

Re-work the previous problems using the relativistic equations.

$V_{AE} = 20$ m/s and $V_{BE} = -30$ m/s

z = 50.000000000000006 m/s

There nothing to be gained by treating things relativistically when speeds are not at least one-hundredth of the speed of light. At lower speeds, the results from classical calculations are virtually the same as the ones using relativistic ones.

Example:

Spaceship A is moving at a velocity 0.50 c relative to observers on Earth, as it chases Spaceship B, which is moving relative to Earth at 0.30 c.

(a) What is the velocity of A as seen by Spaceship B?

$\beta_{AB} = (\beta_{AE} - \beta_{BE}) / (1 - \beta_{AE} \beta_{BE})$
$= (0.50 - 0.30) / [1 - (0.50)(0.30)]$
$= 0.24$
$v_{AB} = 0.24 (3.0 \times 10^8)$
$= 7.2 \times 10^7$ m/s

(b) What is the velocity of B as seen by Spaceship A?

$v_{BA} = - 7.2 \times 10^7$ m/s