Physics 25 Chapter 26 Lenses

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Light Refraction and Convex Lenses

The speed of light in air or vacuum is $c = 3.0 \times 10^8$ m/s. In transparent substances, other than air or vacuum, the speed of light is less than c.

Let v = speed of light in the substance.

Transparent substances have a property called "index of refraction," symbolized as n:

$n = c/v$

Note that n is the ratio of quantities with the same units (m/s); the units cancel, so the ratio is unit-less.

Substance	n = c/v
Vacuum	1.00
Air	1.00
Water	1.33
Glass	1.50

Example: What is the speed of light in glass?

$v = c/n$

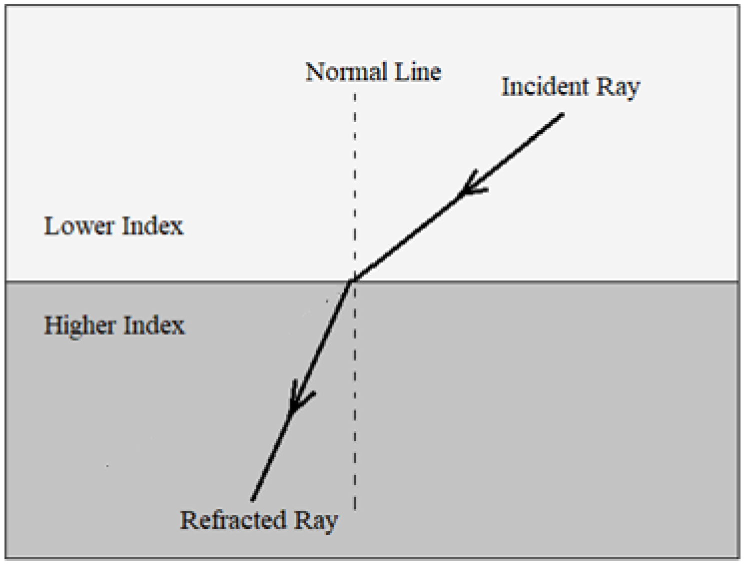
$v = 3.0 \times 10^8 / 1.50$
$= 2.0 \times 10^8$ m/s

If the medium is not stated, "the speed of light" is assumed to be the speed in air or vacuum:

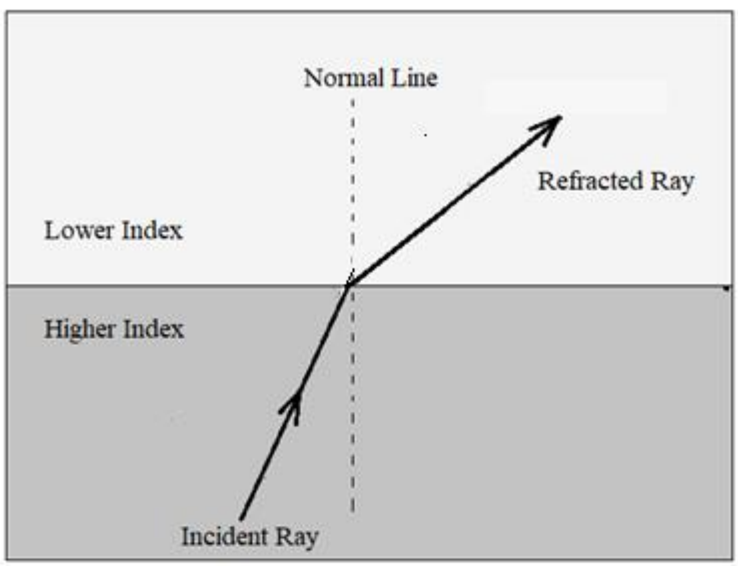
3.0×10^8 m/s

Light Refraction

Refract: to bend or change direction



Light rays entering a medium of *higher* index are bent *toward* the normal line.



Light rays entering a medium of *lower* index are bent *away from* the normal line

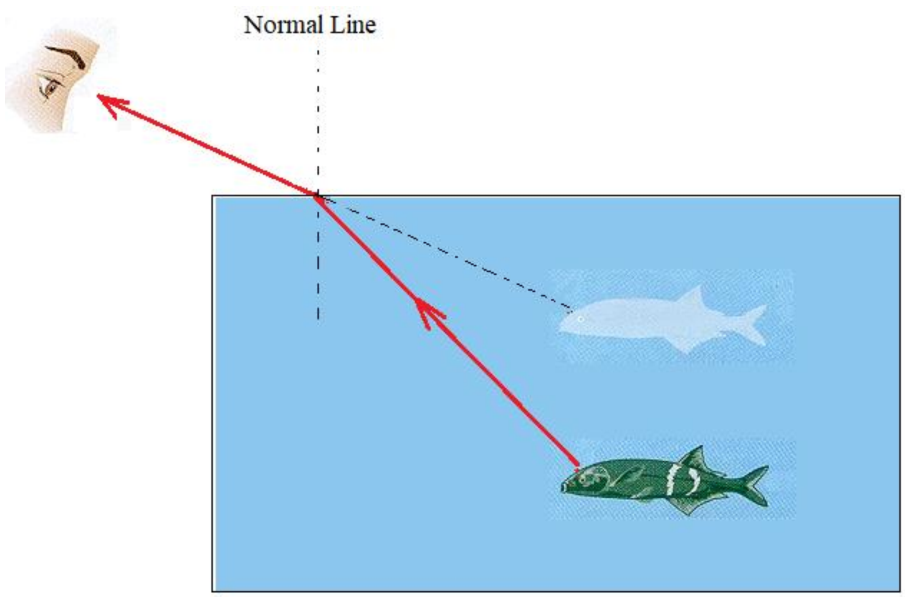
Mnemonic:

Higher-Toward (positive connotations)
Lower-Away (negative connotations)

Apparent Depth Illusion

Objects under water are actually farther below the surface than they seem. The reason for this is given below.

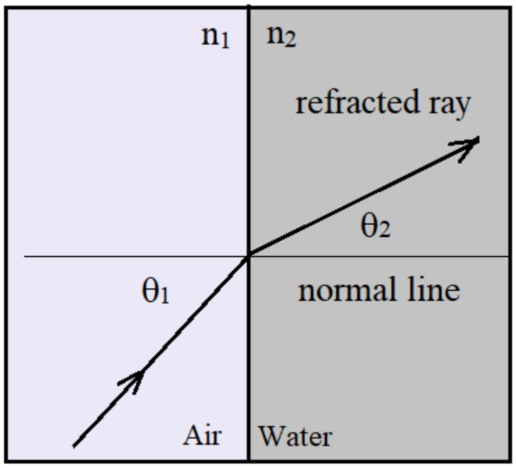
Recall: Light passing from higher to *lower* index is bent *away* from the normal line. The observer's eye extrapolates the ray reaching its eye backward along the broken line to the apparent location of the fish—higher in the water than the fish actually is.



Higher-Toward
Lower-Away

Light reflected off fish travels from higher to lower index, so the light is bent away from the normal, into the eye of the observer.

Snell's Law



$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

θ_1 = angle of incidence
θ_2 = angle of refraction

Transmitted ray enters higher index, so the ray is refracted (bent) toward the normal

<u>Example A</u>:

A ray passing from air to water is incident on the air-water interface at an angle of 24^o. What is the angle of refraction?

$1.0 \sin 24 = 1.33 \sin \theta_2$
$\theta_2 = 17.8^o$

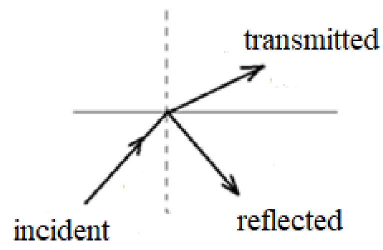
<u>Example B:</u>

What must be the angle of incidence when light travels from water into air if the angle of refraction is to be 90^o?

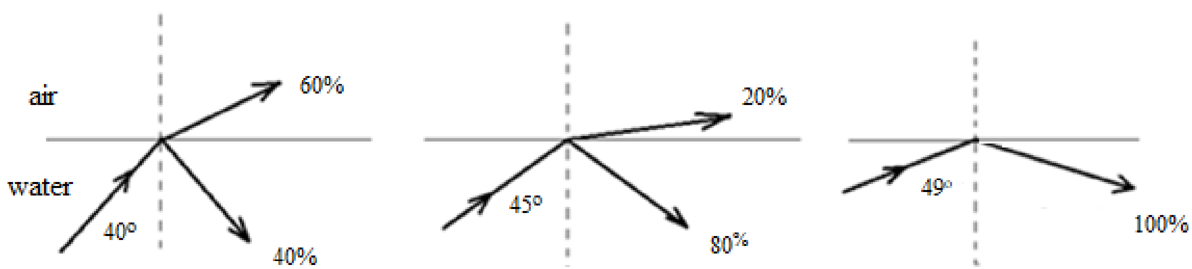
$1.33 \sin \theta_1 = 1.00 \sin 90^o$
$\theta_1 = 48.75^o$

Internal Reflection

In general, part of the light that is incident at an interface between two media is transmitted, while the rest is reflected back into the originating medium, as illustrated below.



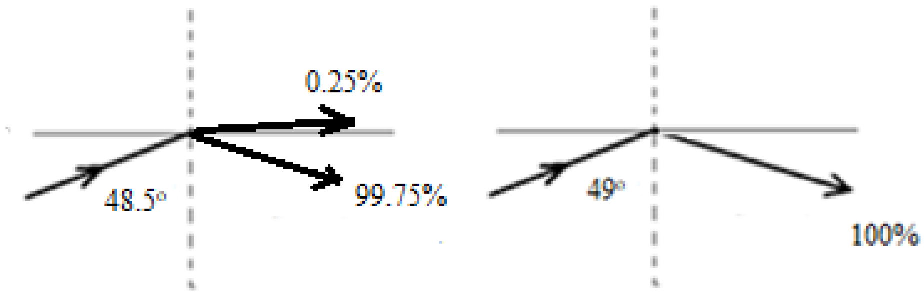
The diagrams below illustrate internal reflection of light at a water-air interface for varying angles of incidence.



The percentage of light energy internally reflected increases as the angle of incidence increases; at or above the "critical angle for total internal reflection," 100% of the light is reflected internally. In the diagrams above, the critical angle is approximately 49°, as calculated below.

Critical Angle for Total Internal Reflection

Total internal reflection occurs when the angle of refraction is 90^o.



The incident angle at which the angle of refraction is 90^o is found using Snell's law:

$n_1 \sin\theta = n_2 \sin 90$
$= n_2$
$\theta_c = \mathrm{Sin}^{-1}\,(n_2/n_1)$

Air: n = 1.00
Water: n = 1.33
$= \mathrm{Sin}^{-1}\,(1.00/1.33)$
$= 48.75^o$

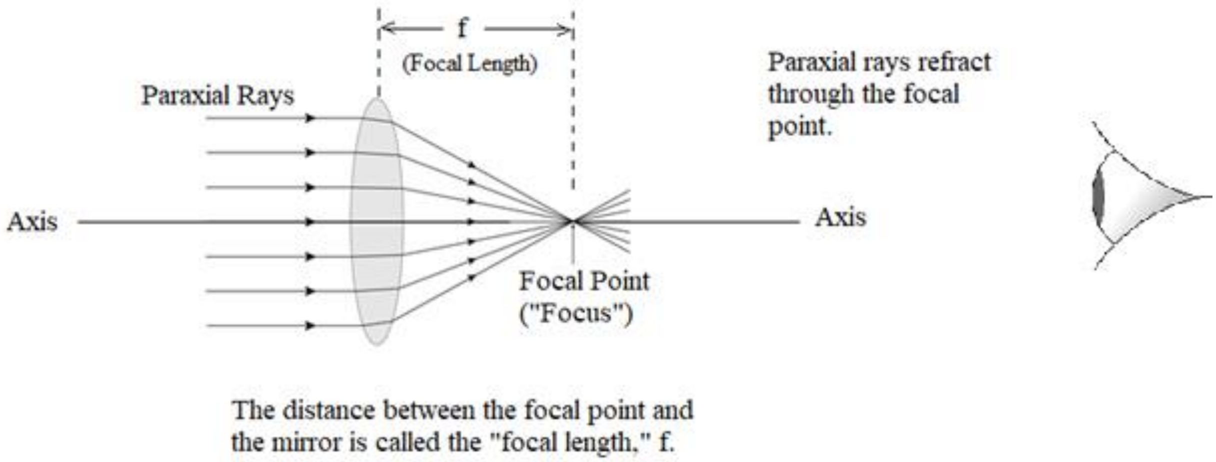
This angle is called the "critical angle" for total internal reflection at a water-air interface.

At any angle of incidence greater than θ_c, no light escapes into air; 100% of the light is internally reflected back into water. At an angle very slightly less than the critical angle, the transmitted light just barely "skims" the surface of the water, and is an extremely small fraction of the incident energy.

Convex Lenses

Two special rays:

1. Paraxial Rays

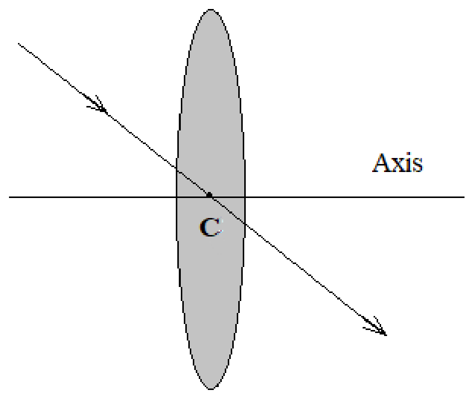


The focal lengths of convex lenses are positive.

Paraxial sunlight passing through a convex lens sets fire to dried twigs located at the lens's focal point.

2. Center Rays

Rays aimed at the center of the lens are not refracted; they go straight through.



Convex Lens Equations

The equations relating to convex lens are *exactly* the same as those for concave mirrors.

Convex Lens Equations:

(1) $1/x + 1/y = 1/f$
(2) $M = - y/x$
(3) $M = f / (f - x)$
(4) $H_I = MH_o$

f: positive
x: positive

If image is not on the eye-side, the image distance y is negative, as illustrated in the figure below.

Object distances are always positive.

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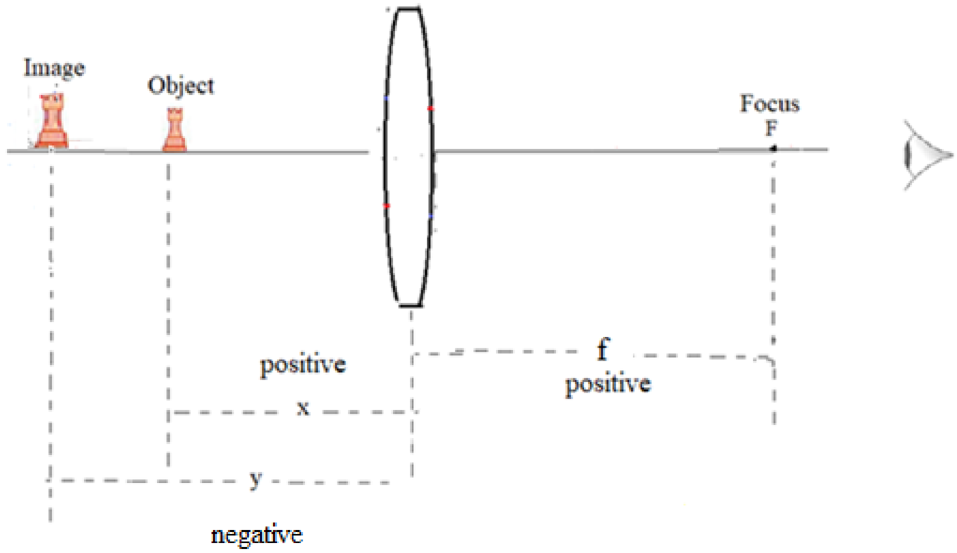
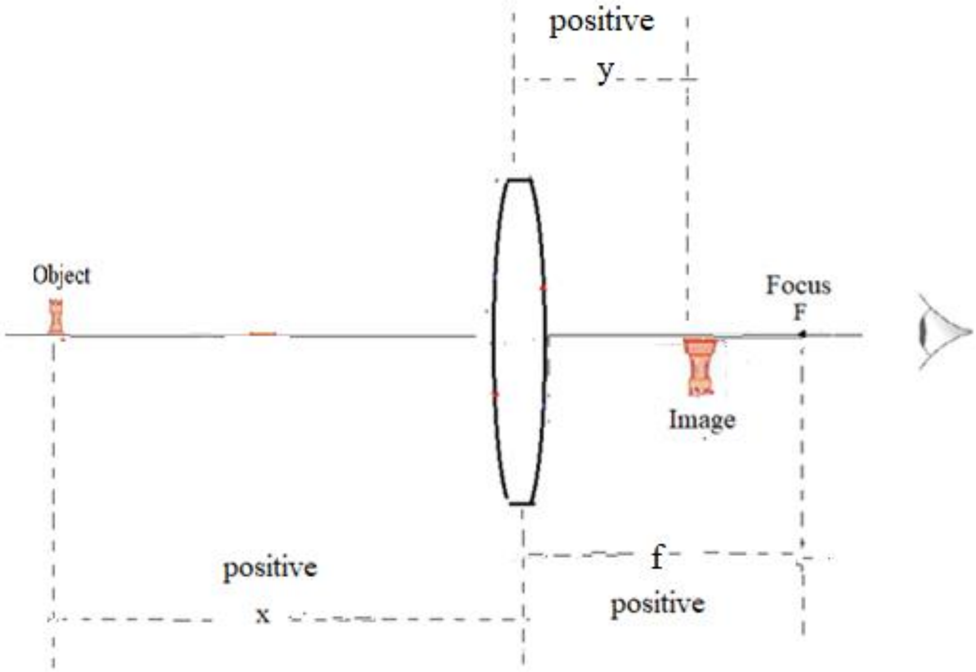
The figure below makes clear the meaning of the variables x, y, and f, in a typical case in which the image is not on the eye-side. In some cases, the image is on the eye-side.

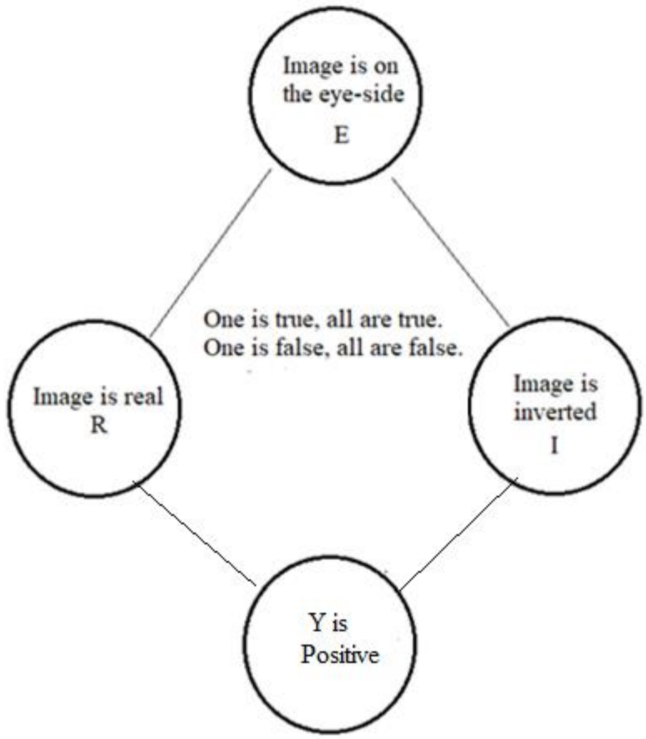


In the figure below is shown a possible image-object-focus configuration; in this case, the image is on the eye-side; its image distance is positive.



The diagram below is the same as the one shown in Chapter 25. It shows that if one of the statements is true, the other three are also true. If one is false, the others are also false.

Mnemonic: YIRE



<u>Example</u>:

The focal length of a convex lens is 20 cm. An object 10-cm tall is placed 15 cm from the lens. What are the attributes of the image?

Recall: Object distances x are always positive, so $x = 15$ cm.

$$M = f / (f - x) = 20 / (20 - 15) = 4$$

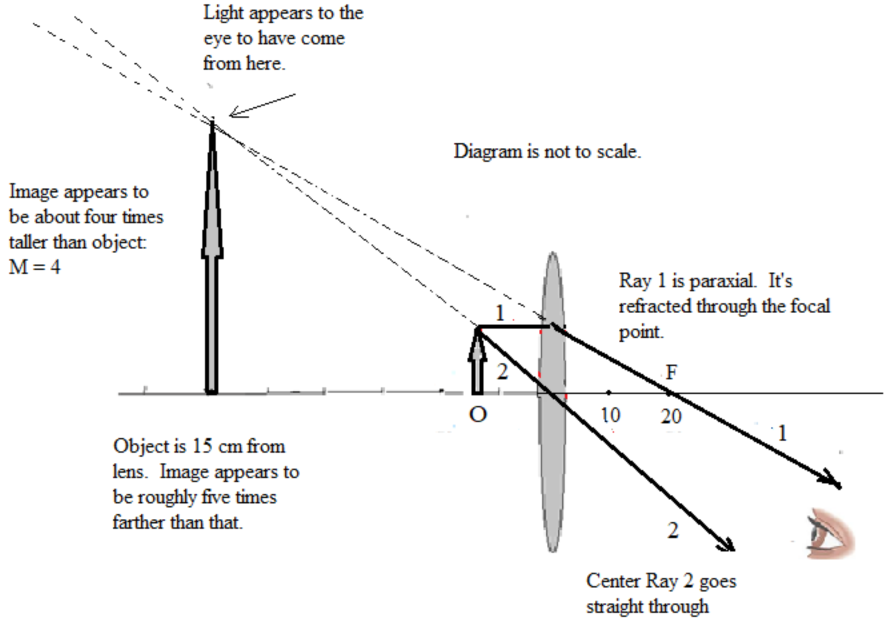
The image is 4 times taller: $H_I = 40$ cm
Use YIRE to determine other attributes:
M is positive, so the image is upright, not inverted (not I). Therefore by YIRE, not I implies not R and not E: The image is not real, and not on the eye-side.

Image Formation with Convex Lenses

The two special rays (paraxial ray and center ray) described earlier can be used to "triangulate" the location, size, and orientation of images formed with convex lenses. The example below illustrates this.

Example:

The focal length of a convex lens is 20 cm. A 10-cm tall object is placed 15 cm from the lens. What is the height of the image, and is it virtual, or real?



Note: the image is about 60 cm from the lens, but it's not on the eye-side, so we apply a negative sign to get y = -60 cm.

The magnification estimated to be about 4.

No light emanates from the image location, so the image is not real (not R). It's also not inverted (not I). An alternative way to conclude that the image is not real is to invoke the YIRE mnemonic rule: if not I, then not R.

Let's use the lens equations to see how closely the exact values of y and M compare to the estimate ones we found above.

Example:

In the example above a 10-cm tall object is 15 cm from a convex lens whose focal length is 20 cm. From the diagram we estimated that M = 4, and that the image is 60 cm from the lens.

Note: Images that are not on the eye side have negative image distances:

$y = -60$ cm

Use the lens equations to determine the exact values of the magnification M and the image distance y.

$f = 20$ cm
$x = 15$ cm

Calculate the Image Distance
$1/15 + 1/y = 1/20$
$y = -60$ cm (same as estimated from the figure)
Calculate the Magnification
$M = -y/x$
$= -(-60)/15$
$= 4$ (same as estimated)

Example:

The focal point of a convex lens is 20 cm. A 2.0-cm tall object is 40 cm from the lens.

Find the position, height and orientation of the image.

Calculate Image Distance

$1/x + 1/y = 1/f$

$1/40 + 1/y = 1/20$

$y = 40$ cm

Note: y is positive, so the image is on the eye side, and therefore real.

Calculate Magnification

$M = -y/x$

$= -40/40$

$= -1$

Alternatively, using the other M equation:

$M = f / (f – x)$

$= 20 / (20 – 40)$

$= -1$

M is negative, so the image is inverted.

Calculate the image height

$H_I = MH_o$

$= (-1)(2.0)$

$= -2.0$ cm

Example:

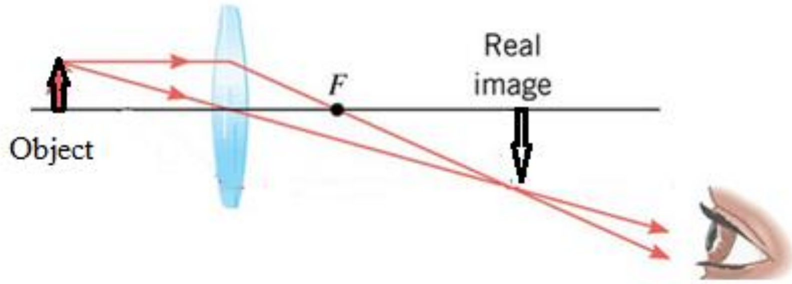
Prove that if an object is between the focal point and the lens, i.e., $f > x$, the image is *upright* and *taller* than the object

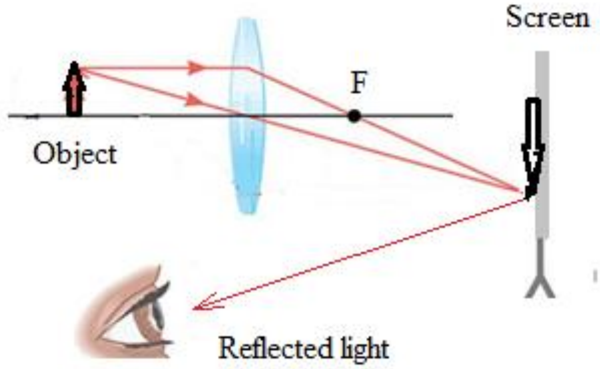
$M = f / (f – x)$
= pos / (pos – smaller pos)
= pos/smaller pos
= pos > 1

M is positive, and greater than 1.0, which means the image is upright, and taller than the object.

Slide Projector Application

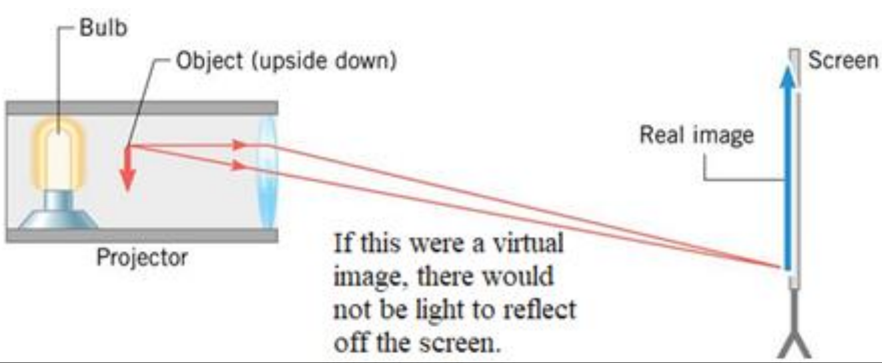
This example will illustrate the importance of knowing if images are real, or virtual.



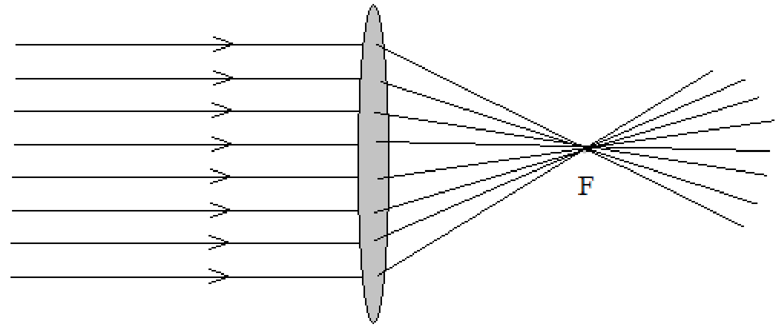


If there were not really light at the image location--i.e., if the image were not real, but instead virtual, there would be no light to reflect off the screen into the viewer's eyes.

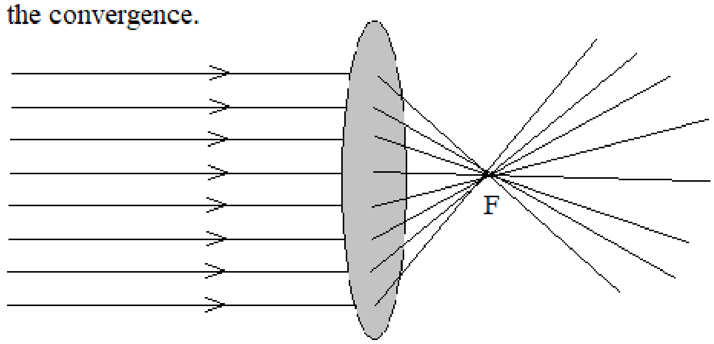
Note that the image on the screen is inverted. To obtain an upright image with a slide projector, the object slide must be upside-down in the projector.



Lens Power



The smaller the focal length, the greater the reciprocal, 1/f, called the "lens power." The greater the lens power, the more severe is the convergence.



The greater the curvature of the lens, the greater is the lens's convergence power, i.e., the closer to the lens is the focal point, i.e., the shorter is the focal length, f.

<table>
<tr>
<td>Lens power is also called “diopter power,” and “refractive power.”

$D = 1/f$
Units: m^{-1} (Note: meters, not centimeters.)
1.0 “diopter” (Dp) = 1.0 m^{-1}

<u>Example</u>:

(a) What is the diopter power of a lens having a focal length of 50 cm? (b) What is the diopter power if the focal length is 40 cm?

(a) $D = 1/f$
$= 1/ (0.50\ m)$
$= 2.00\ m^{-1}$
= 2.00 Dp

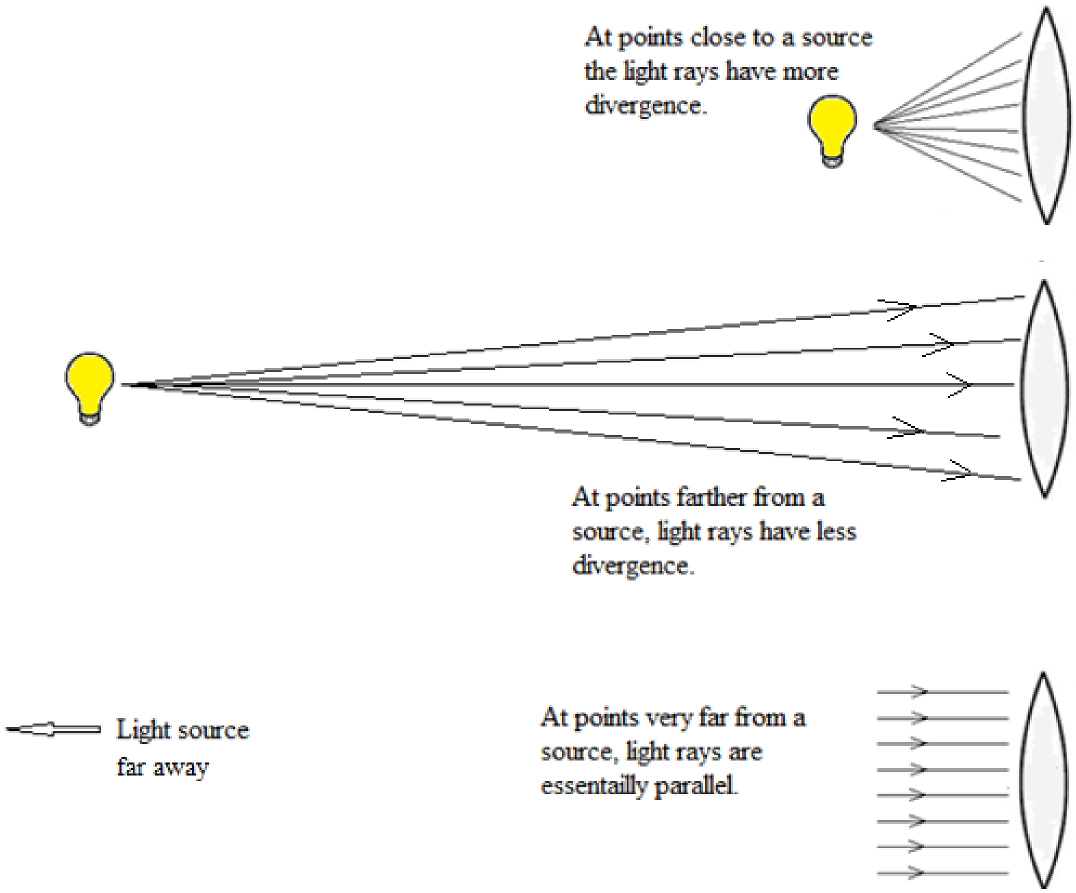
(b) $D = 1/f$
$= 1/ (0.40\ m)$
$= 2.5\ m^{-1}$
= 2.5 Dp</td>
<td>

The lens power of “reading glasses” sold in drug stores is usually printed on the inside of one of the temples.</td>
</tr>
</table>

In humans, the total optical power of the relaxed eye is approximately 60 diopters. The cornea accounts for approximately two-thirds of this refractive power (about 40 diopters) and the crystalline lens contributes the remaining one-third (about 20 diopters). Reading glasses usually range in diopter power from 1.00 to 3.00 Dp.

Far-Sightedness

The lenses of far-sighted persons are unable to cause sufficient convergence of the diverging rays that are incident on the eyes' lenses from objects close to the eyes. Light rays from objects that are relatively closer to the eyes have strong divergence at the eyes, while rays from distant objects are far less divergent, and even almost parallel. The figures below illustrate this:



The lenses of farsighted persons don’t do enough converging of rays from nearby objects.

To make up for this deficiency, the reader needs to hold the newsprint farther from the eye than a person with normal vision does in order that the amount of divergence of the rays incident on the newsprint is lessened.

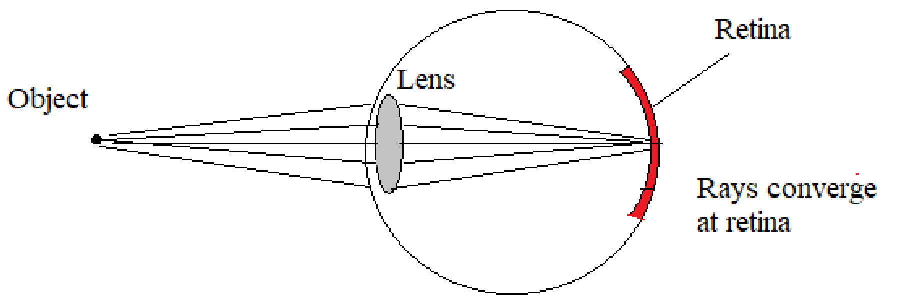
As persons age, their farsightedness usually increases, which means the distance from the eyes at which a book must be held gets greater with each passing year.

The nearest distance from the eyes a person can read newsprint without effort is called the “near point.” In the cartoon at the right, the “farsighted” person’s near point appears to be more than an arm’s-length away from his eyes.

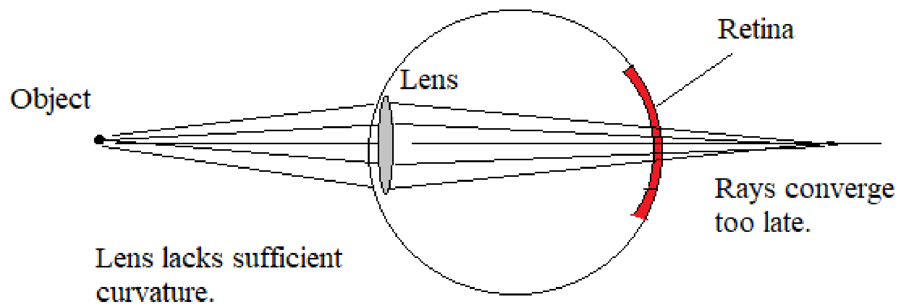
In order to read newsprint close to the eye, the farsighted person must wear lenses that create images of newsprint that are no closer to the reader than his near point.

The remedy for farsightedness is to place in front of the eyes a converging lens to make up for the under-convergence of the eye lenses. This is illustrated below.

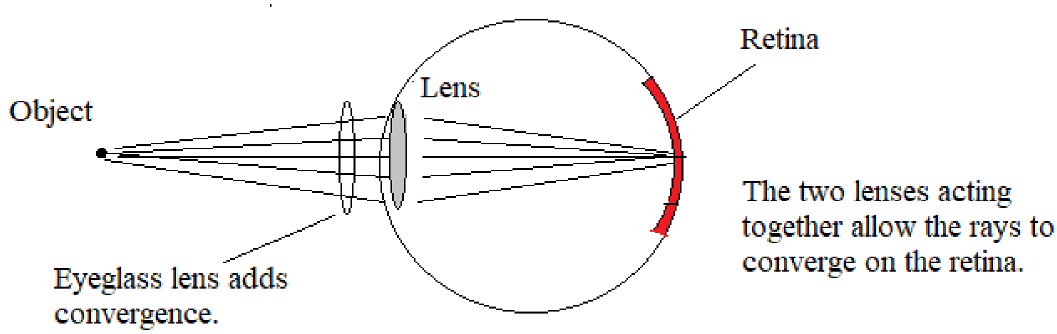
Normal Eye



Farsighted Eye



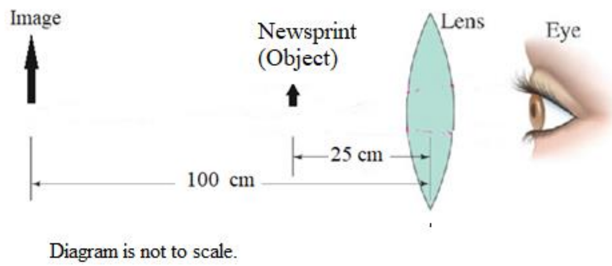
Farsighted Eye



Example:

A person's near point is 100 cm from her eyes. She wishes to read newsprint 25 cm from her eyes.

Ignoring the thickness of the lens and the distance between the eye-glasses and her eyes, what must be the lens power (in diopters) of eyeglasses that would form an image at her near-point of newsprint 25 cm from her eyes?



x = 25 cm

Note: The newsprint image is *not* on the eye-side, so y is negative:

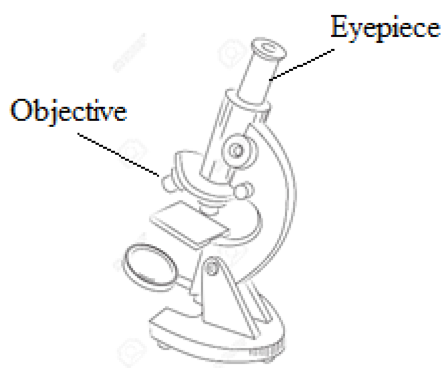
$$y = -100 \text{ cm}$$
$$x = 25 \text{ cm}$$
$$1/x + 1/y = 1/f$$
$$1/25 - 1/100 = 1/f$$

$$f = 3.00 \text{ m}^{-1}$$
$$= 3.00 \text{ diopters}$$

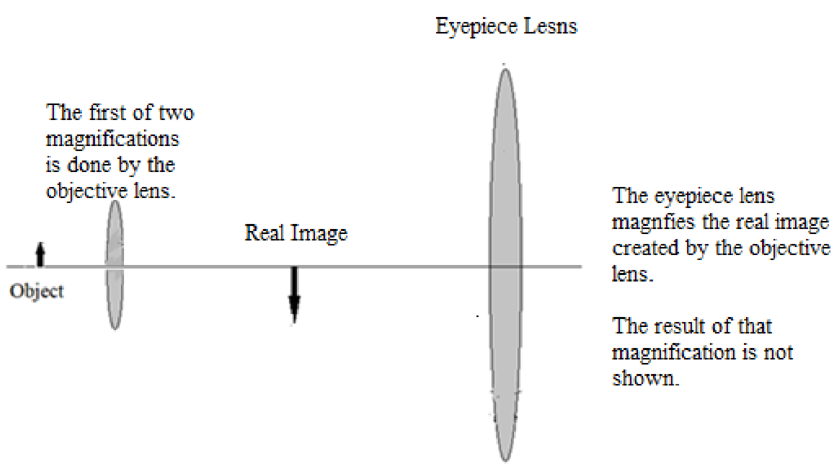
The Compound Microscope

A compound microscope uses two lenses to obtain a view of an object that has a greater magnification than is possible with just one lens.

The lens that is closest to the object is called the *objective*. The image formed by the objective is has attributes determined by the lens equations. The result (see below) is a real image, which serves as the object viewed by the eyepiece. The importance of knowing the image formed by the objective is real is that real images can be magnified, while virtual ones cannot.



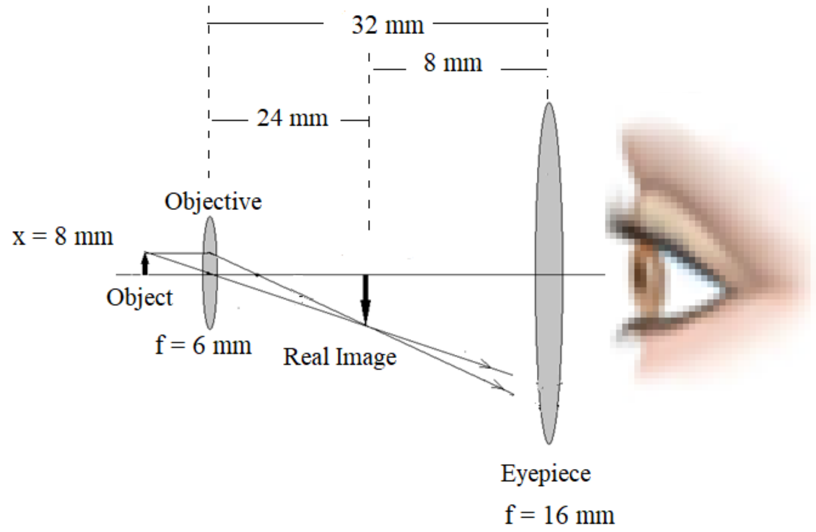
The image formed by the objective lens below serves as the "object" viewed by the *eyepiece*, as shown below.



The overall magnification is the product of the magnifications achieved by the objective lens and the eyepiece. The examples below illustrate this procedure.

Example

The objective and eyepiece lenses of a microscope are 32 mm apart. The focal length of the objective is 6 mm, and the object is 8 mm from the objective. The focal length of the eyepiece is 16 mm.



The final image, the result of the magnification of the real image, is not shown.

Objective Multiplication

$1/8 + 1/y = 1/6$

$y = 24$

$M = 6/(6 - 8)$

$= -3$

The real image created by the objective lens serves as the object for the eyepiece. The object distance is given below and shown above:

$x = 32 - 24$

$= 8$ mm

Eyepiece Multiplication

$M = f/(f - x)$

$= 16/(16 - 8)$

$= 2.0$

The overall magnification is the product of the two magnifications:

$M = 2.0\ (-3)$

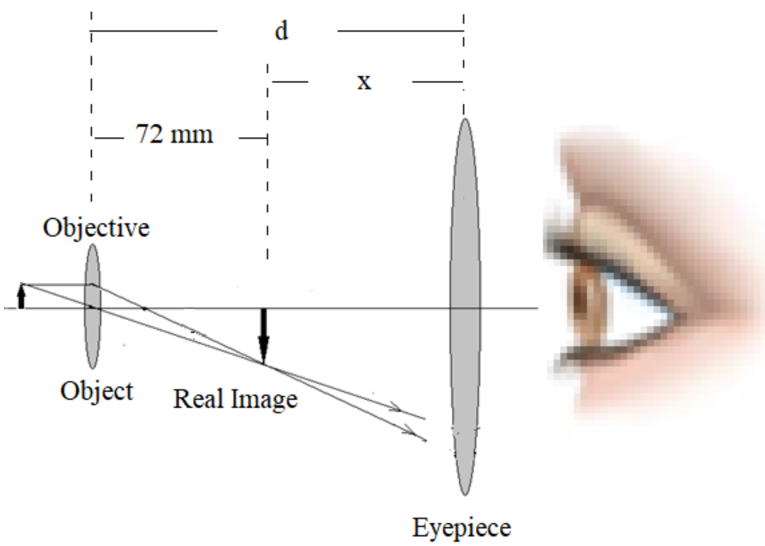
$= -6.$

Example:

The objective and eyepiece focal lengths of a compound microscope are 8 mm and 40 mm, respectively. What must be the distance d between the two lenses in order that an object placed 9 mm from the objective has an overall magnification M = -80?

Magnification by Objective

$$1/9 + 1/y = 1/8$$
$$y = 72 \text{ mm}$$
$$M = -72/9$$
$$= -8$$



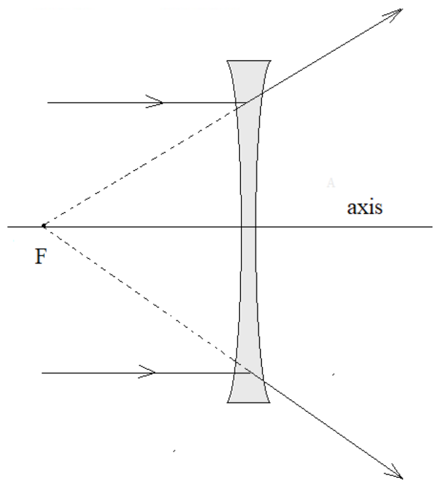
Magnification Due to Eyepiece

The product of the two multiplications must be -80:

$$-8\,M = -80$$
$$M = 10$$
$$M = f/(f - x)$$

$$10 = 40/(40 - x)$$
$$x = 36 \text{ mm}$$
$$d = x + y$$
$$= 36 + 72$$
$$= 108 \text{ mm}$$

Concave Lenses



Paraxial rays, after passing through concave lens, seem to have originated at the focal point, F.

Rays aimed at the center go straight through.

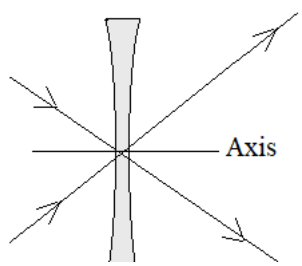
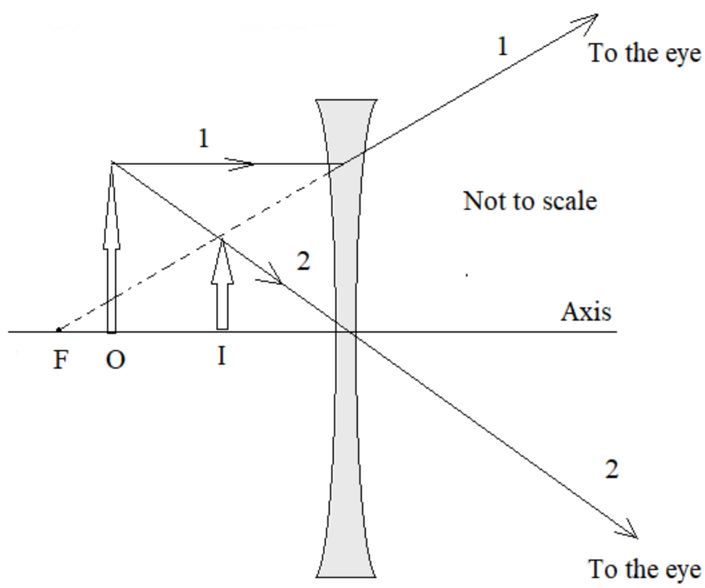


Image Formation with Concave Lenses

Image is smaller, upright, and virtual.



Concave lens image formation follow the same rules, and same equations as for convex lenses, except the focal length for concave lenses are *negative*.

Problems involving concave lenses are solved exactly like the problems we solved in Chapter 25 for convex mirrors which, like concave lenses, have negative focal lengths.