Physics 25 Chapter 25 Mirrors

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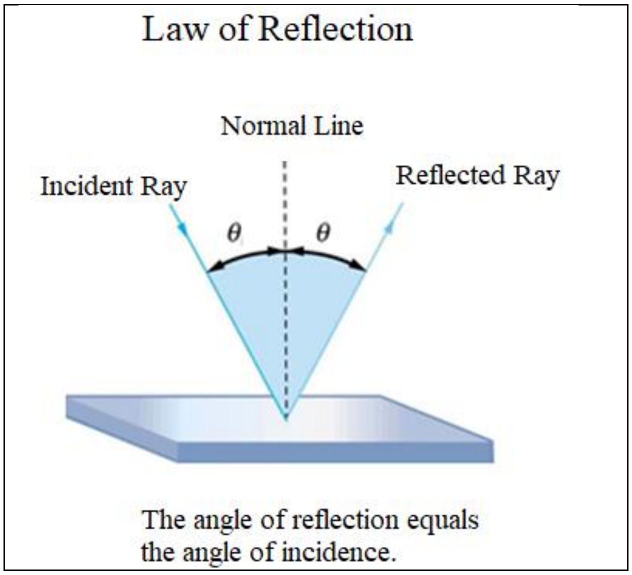

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<u>Example</u>:

The sides of two mirrors, M_1 and M_2, perpendicular to the plane are aligned as shown in the figure. A ray of light is incident on M_1 from the left and is reflected onto M_2.

What is the angle θ?

Answer: 70°
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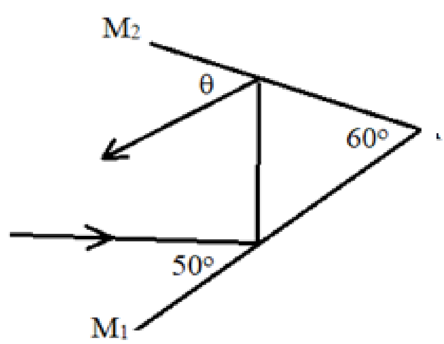

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Image Formation with Plane Mirrors

A light source is illuminating the chess-piece castle in Figure 1 below. Rays 1 and 2 emanating from the top of the object reflect off the mirror according to the reflection law and are directed toward the eye. Figure 2 illustrates the adherence to the reflection law for each of the two rays: the angle of incidence equals the angle of reflection.

The eye-mind system extrapolates Rays 1 and 2 back to their intersection point in back of the mirror; the intersection point is the location of the top of the image. Other image points (not shown) corresponding to the bottom, middle, and other points of the castle would be determined in a similar manner.

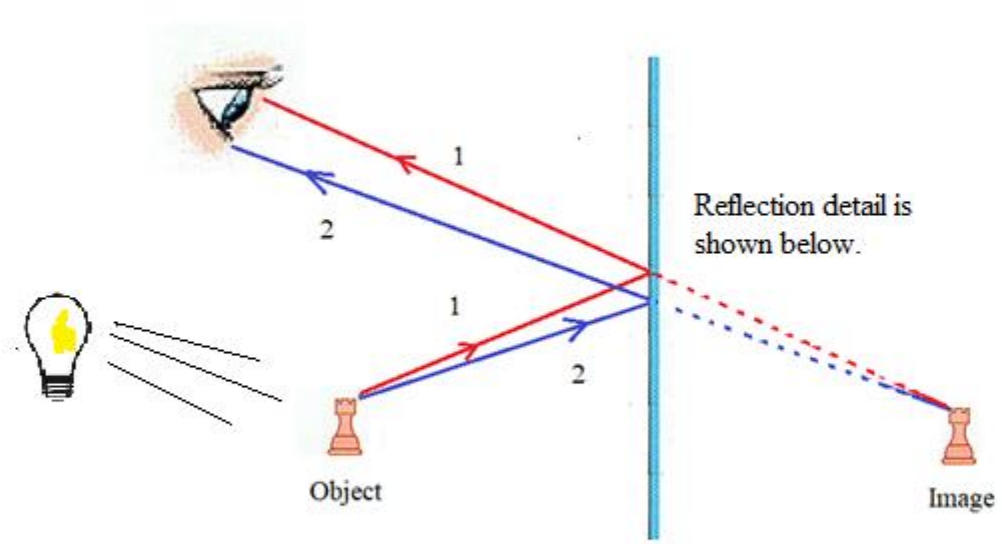


Figure 1

The figures below illustrate the Law of Reflection applied to Rays 1 and 2:

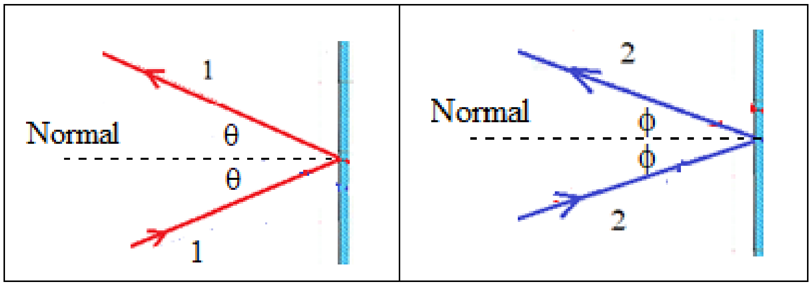


Figure 2

Note that the castle image is in <u>back</u> of the mirror, so light rays that reach the eye only *appear* to be coming from the image. Images from which light rays are not *really* coming, are said to be "virtual."

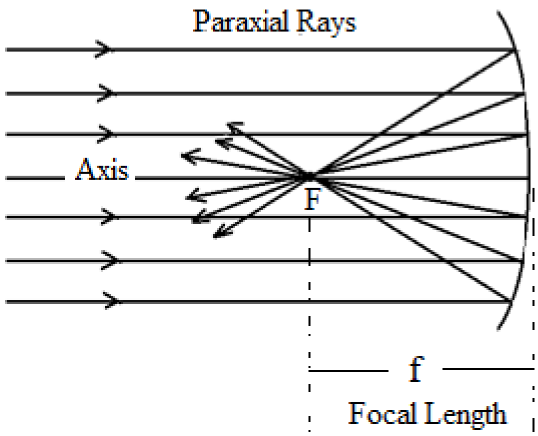
Concave Mirrors

Creation of concave mirror image diagrams requires a knowledge of the behavior of two special rays, the first one of which is described below.

Special Ray #1: Paraxial Rays

Incident rays that are parallel to the axis are called “paraxial rays.” These rays are reflected through a common point on the axis at a point called “the focal point,” F, also called “the focus.”

The distance of the focal point from the mirror is called, “the focal length.” Concave mirror focal lengths are positive.



Paraxial ray sunlight is focused on the cooking pot at the focus.

Special Ray #2: Focal Point Ray

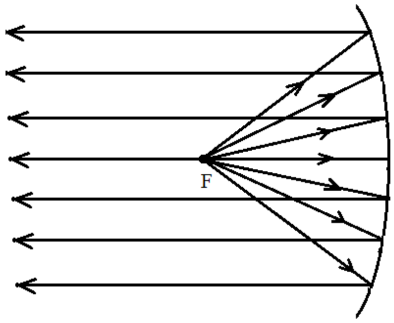
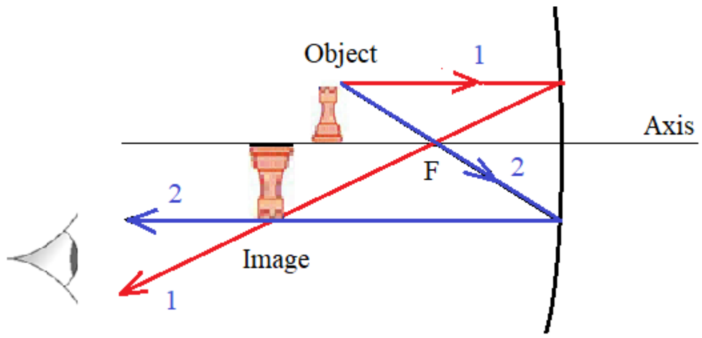
Focal point rays emanate from the focal point on their way to the mirror, where they are reflected paraxially.  	A light source placed at the focal point of a concave mirror produces a parallel beam of light. Photo below shows a searchlight in operation in Great Britain in World War II. 

Image Formation with Concave Mirrors

A light source (not shown) illuminates the castle chess-piece resting on the “axis” below. Rays 1 and 2 of light reflected off the top of the castle travel in the directions indicated.



Ray 1 is a paraxial ray: it reflects through the focal point.

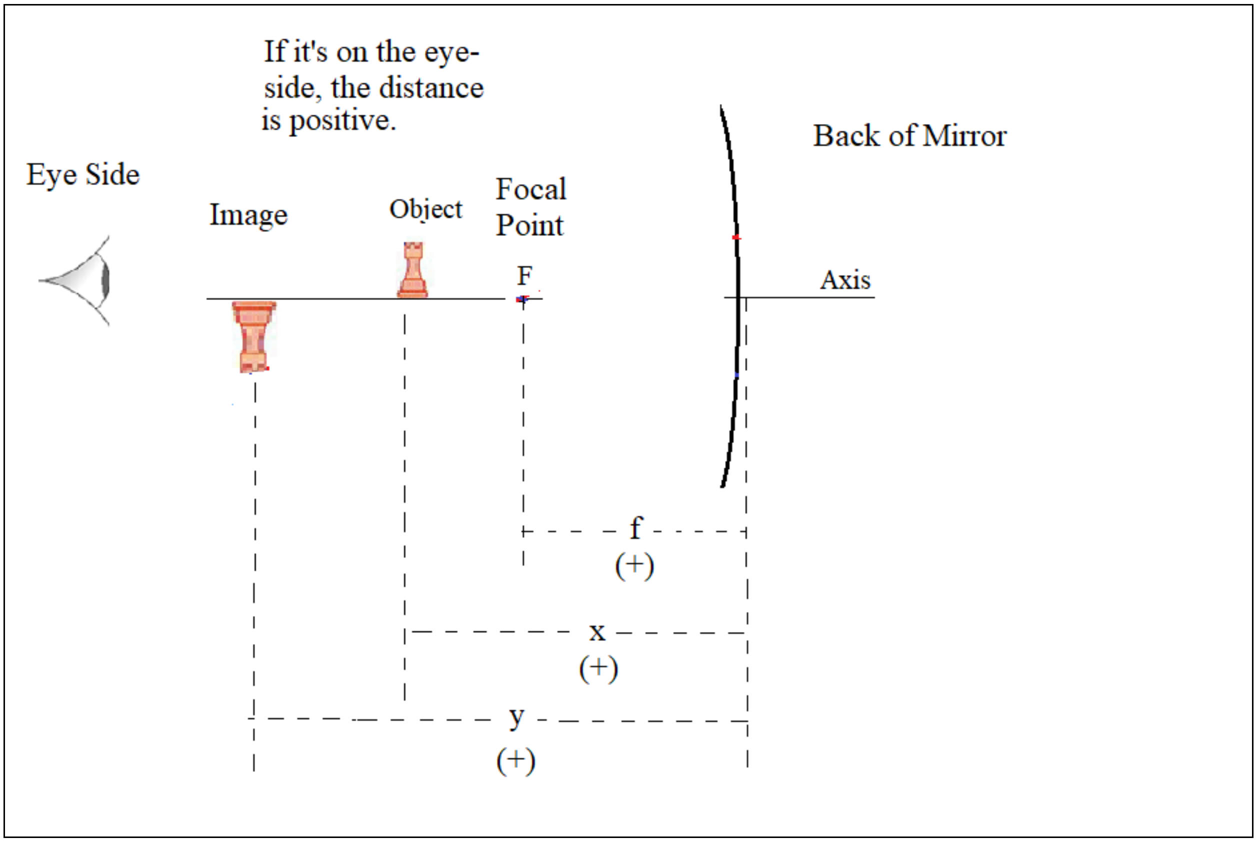
Ray 2 is a focal point ray: after passing through the focal point it reflects paraxially.

The eye-mind system uses Rays 1 and 2 to extrapolate backward to their intersection point at the bottom of the chess-piece; to the uninformed mind, that is the apparent location and orientation of the chess-piece. Other chess-piece points are determined by a similar extrapolation procedure. The image in this case is inverted, and is below the axis.

We earlier noted that images that appear to come from behind the mirror are not real, but “virtual.” Light does not really emanate from such images. In this case, light really does travel from the image to the eye. Such images are called “real.”

Any image that is on the eye-side, not in back of the mirror, is real.

Concave Mirror Equations



Example above: All positive: f = Focal length x = Object distance y = Image distance

H_I = Image Height Negative if inverted.	H_o = Object Height Always positive	M = Magnification = H_I/H_o Negative if inverted.

<u>Mirror Equations</u>:

(1) $1/x + 1/y = 1/f$
(2) $M = - y/x$
(3) $M = f / (f – x)$
(4) $H_I = MH_o$

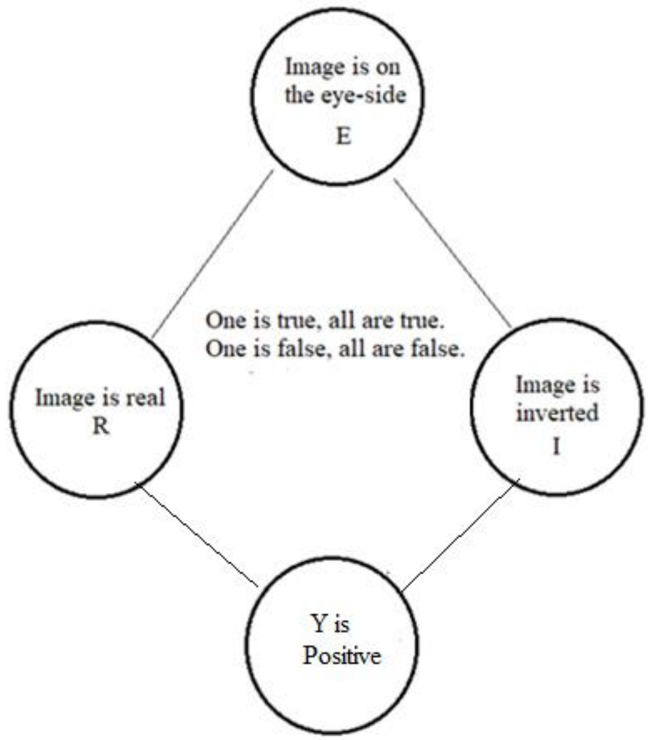
Eqn. (1) is sometimes called “the reciprocals equation.”

Facts and Examples: Concave Mirrors

Objects: are always on the eye-side (in front) a distance x away from the mirror. "Object distances" x, are always positive.
Images: Images on the "eye-side" (in front) of the mirror of the mirror are a distance, y, from the mirror. The image distance is negative if the image is in back (behind) the mirror. Images in back of the mirror (y is negative) are not real (they're "virtual).
Focal points are always in front of the concave mirror. Their distance from the mirror is called the "focal length," which is a positive number
Image Multiplication: $M = - y/x$ and $M = f / (f - x)$ If M is positive, image is upright. If M is negative, image is inverted. If $M > 1.0$, the image is taller than the object. If $M < 1.0$, it's shorter.
Object and Image Heights: $H_I = M\ H_o$. Inverted images have negative H_I. Object heights H_o are always positive.
$1/x + 1/y = 1/f$ If any two of the distances x, y, and f, are known, the reciprocals equation determines the third distance. Other equations can accomplish the same thing. If y and M are given, for example, x can be found from $M = -y/x$.

If any one of the following statements is true, the other three are likewise true; otherwise, if one is false, they all are false.

Mnemonic: YIRE



For example, if f = 20 cm and x = 10 cm, then from the “reciprocals” equation, we get

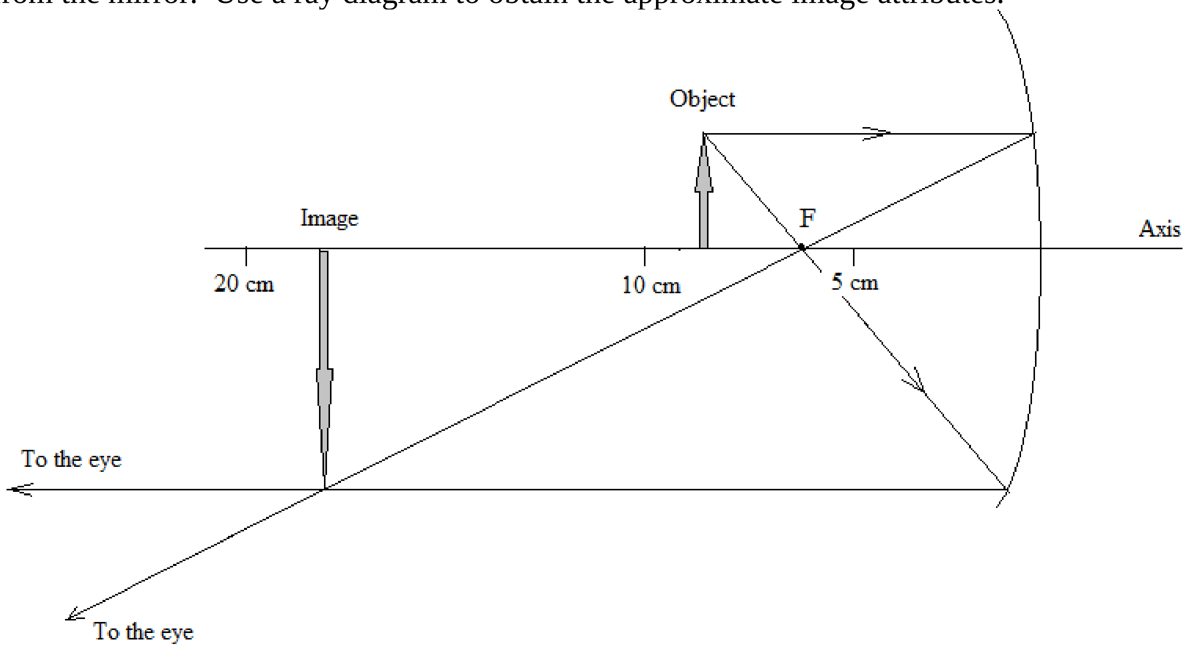
$$1/10 + 1/y = 1/20:$$

$$y = -20 \text{ cm}$$

By the YIRE mnemonic, y is not positive so the image is not real, not inverted, and not on the eye side.

Example A:

The focal length of a concave mirror is 6 cm. An object (represented as an arrow) is 9 cm from the mirror. Use a ray diagram to obtain the approximate image attributes.



We estimate that the image is about 18 cm from the mirror (x = 18 cm), on the eye side. The image is inverted, and appears to be about twice as tall as the object and inverted, so we estimate that the magnification is about M = -2.

Three ways to know the image is real:

1. Image is inverted (I), so by YIRE the image is real.
2. The image is on the eye-side (E), so also by YIRE it's real (R).
3. Light rays really do emanate from the image, so it's real.

Next, let's use the mirror equations to object exact values of the image location (y) and magnification (M).

Example A:

Use the mirror equations to obtain the attributes of the image formed in the previous example.

$f = 6$ cm
$x = 9$ cm

$1/9 + 1/y = 1/6$
$y = 18$ cm
(Using ray-diagraming we earlier estimated y was 18 cm.)
$M = -y/x$
$= -18/9$
$= -2$
(We estimated $M = -2$.

Example B:

The focal length of a concave mirror is f = 6 cm. A 4.0-cm tall object is placed 3 cm from the mirror. Determine the image attributes.

$M = f / (f – x)$
$= 6/(6 – 3)$
$= 2$
$H_I = 2 H_o$
$= 8.0$ cm
M is positive, so the image is upright, not inverted, not real and not on the eye-side.

$1/3 + 1/y = 1/6$
$y = -6$ cm
6 cm in back of the mirror

<u>Example A:</u> Prove that objects located between the focal point and the concave mirror (f > x) always have virtual, upright images taller than the object. Recall: the focal lengths of concave mirrors are <u>positive</u>. $M = f / (f - x)$ $= pos / (pos\text{-}smaller\ pos)$ $= pos > 1$ The image is not inverted, so by I is not true, so E is not true and R is not true: Image is not on the eye side, and the image is not real. M is greater than 1, so the image is taller.	 Object hand is located between the mirror and the focal point; as proved at the left, the image is upright and taller.

<u>Example B</u>:

The focal length of a concave mirror is 40 cm. How far from the mirror must an object be placed in order that its image be upright and four times taller than the object?

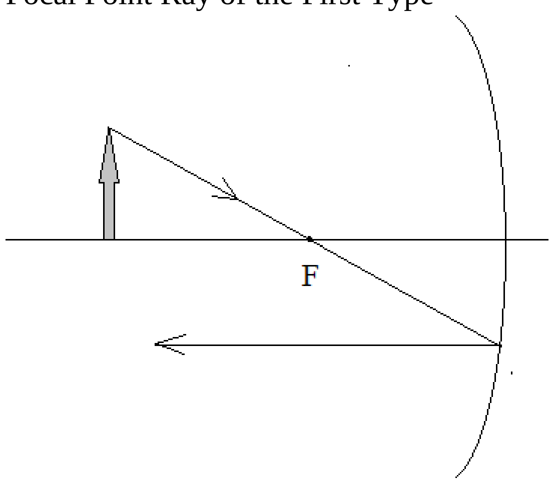
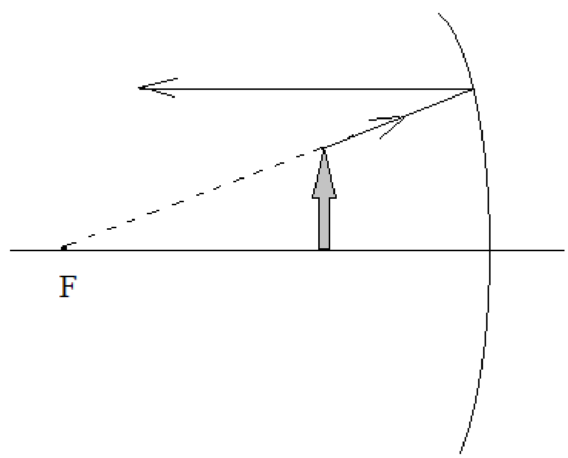
$$-y/x = 4$$
$$y = -4x$$
$$1/x - 1/4x = 1/40$$
$$3/4x = 1/40$$
$$x = 30 \text{ cm}$$

Example:

Use YIRE mnemonic to prove that one cannot form an upright real image with a concave mirror: if one is not true, the other three are also not true.

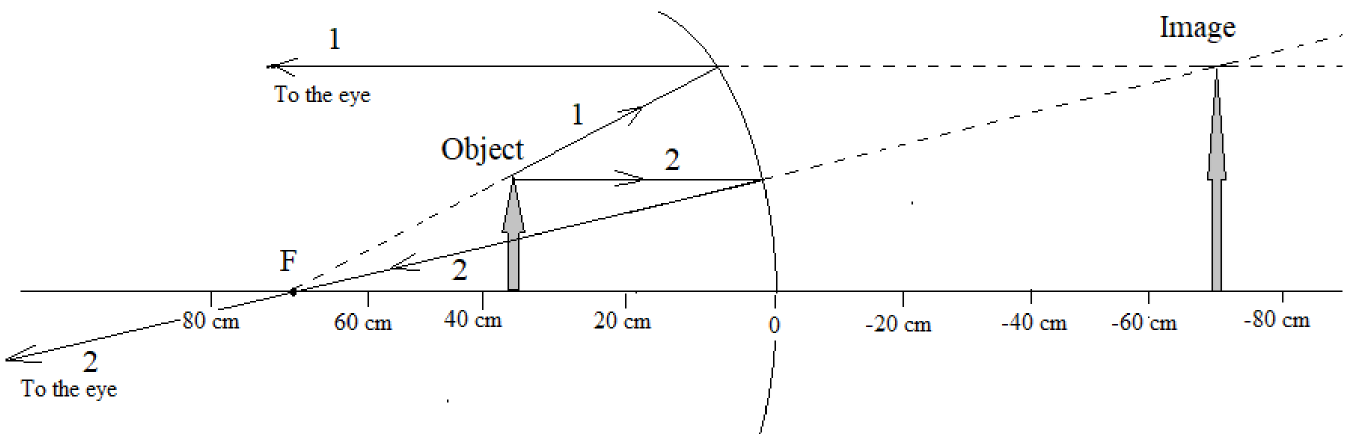
Proof: If the image is upright, it's not inverted (not I), so the image is not real R and not on eye side (E) and y is not positive.

A Second Type of Focal Point Ray for Concave Mirrors

Focal Point Ray of the First Type	Focal Point Ray of the Second Type
	
This type of focal point ray was discussed earlier. A ray from the object passes through the focal point on its way to the mirror, and is reflected paraxially.	A ray heading from the object toward the mirror along a path that extrapolates back to the focal point is reflected paraxially. The example below illustrates its use.

Example:

An object is 36 cm from a concave mirror whose focal length is 72 cm. Use a ray diagram to determine the approximate attributes of the image.



We estimate from the diagram above that the image is about double the height of the object, and about 70 cm from the mirror, behind the mirror.

The image is not real because rays of light above don't actually travel from the image, through the back of the mirror, to the eye.

Example:

Use the mirror equations to find the location and magnification of the image in the previous diagram. Recall the data from that example: $f = 72$ cm and $x = 36$ cm.

$$1/36 + 1/y = 1/72$$
$$y = -72 \text{ cm}$$

(We earlier estimated $y = -70$ cm.)

The image distance is negative, so the image is behind the mirror and therefore not real.

$$M = -(-72)/36$$
$$= 2$$

The magnification is 2, so the image is twice the height of the object. We earlier estimated from the diagram that the image was twice the height of the object.

Note that the diagram above is not to scale: If it were drawn to scale, the two rays (1 and 2) heading to the eye would be nearly parallel, and close together, side-by-side.

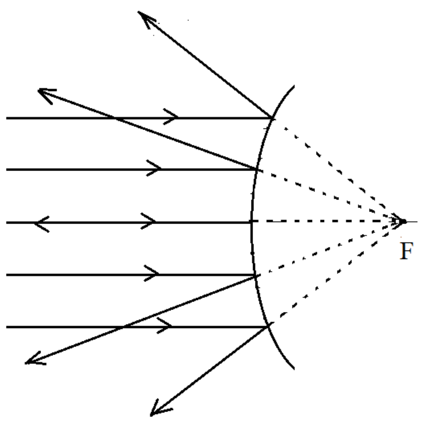
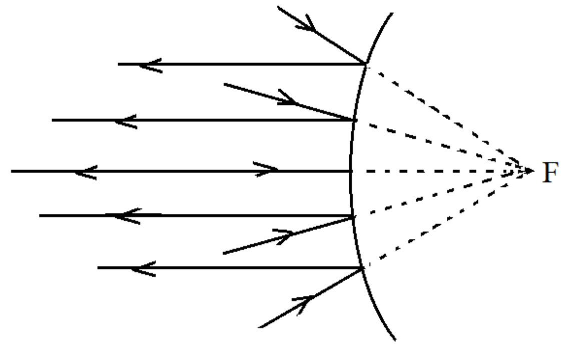
Convex Mirrors

Security Mirrors

Stores use convex mirrors because their bowed-outward shape allows a wider field of view than do similar-sized plane mirrors, as well as the fact that the image sizes are smaller, which allows more objects and persons to be included in the image field.

Special Rays

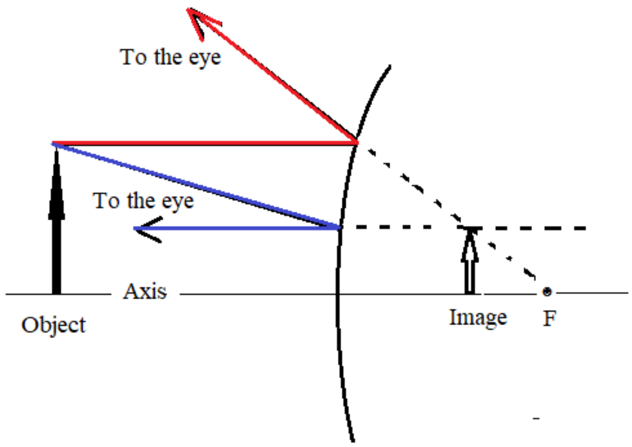
Creation of convex mirror image diagrams requires a knowledge of the behavior of two special rays, described below.

Paraxial Rays, after reflection, appear to have originated behind the mirror at a point called the “virtual” focal point, F. 	**Focal Point Rays** aimed at the virtual focal point are reflected paraxially. 

Notice that the virtual focal point is behind the mirror, and recall rule provided long ago: If the focus (or the object, or the image) is behind the mirror, the length (or distance) is negative. Convex mirror focal lengths are <u>negative</u>.

Example:

What are the characteristics of the image below?



Three ways to know that the image is not real (virtual):

1. Light rays reaching the eye are not emanated from the image. They cannot have traveled from the image through the back of the mirror to the eye, so the image is virtual.

2. By YIRE, because the image is not inverted (I), it's not real (R).

3. By YIRE, because the image is not on the eye side (E), it's not real (R).

The image is upright, and taller.

Note that the diagram above is not to scale: If it were drawn to scale, the two rays heading to the eye would be nearly parallel and much closer together.

The equations below for convex mirrors are *exactly the same* as that given for concave mirrors. However, the student needs to know that convex mirrors have negative focal lengths.

Convex Mirror Equations

(1) $1/x + 1/y = 1/f$
(2) $M = - y/x$
(3) $M = f / (f – x)$
(4) $H_I = MH_o$

These equations are the same as those used for concave mirrors.

Example:

Prove that convex mirrors cannot form inverted images.

Recall: the focal length f of convex mirrors is negative, and the object distance x for all mirrors is always positive.

$M = f / (f – x)$
$= (neg) / (neg – pos)$
$= (neg) / (neg)$
$= pos$

Convex mirror images are always upright, never inverted.

Example A:

Prove that images formed with convex mirrors are always shorter than the object, i.e., show that M is always less than 1.

Recall: f is negative for convex mirrors, and the object distance, x, for all types of mirrors is always positive.

Recall:

$M = f/(f - x)$
$= (\text{neg}) / (\text{neg} - \text{pos})$
$= (\text{neg}) / (\text{more neg})$

The denominator is more negative than the numerator f, so M <1.

Upright *shorter* images formed with convex mirrors give the false impression that the object is farther from the mirror than it actually is.

This explains why the warning written on passenger-side convex mirrors states,

"*Objects in mirror are closer than they appear*."

Example B:

The focal length of a convex mirror is -12 cm. How far from the mirror may an object be placed in order that the magnification be 0.25?

$$M = f / (f - x)$$
$$0.25 = -12 / (-12 - x)$$
$$x = 36 \text{ cm}$$