Physics 25 Chapter 22 Faraday’s Law Lenz’s Law

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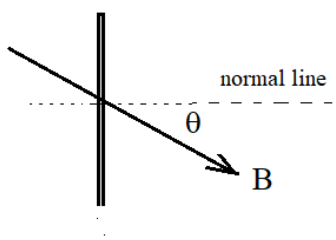
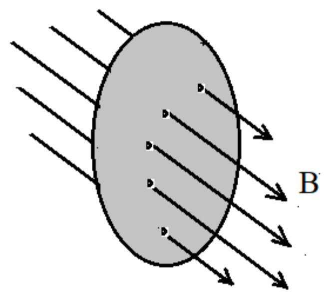
Magnetic Flux

Side view of loop



A = area



$$\Phi = BA\cos\theta$$

Units: tesla-meter^2 (T-m^2)

Note: A “normal” line is perpendicular to a plane.

Flux Change

Magnetic flux will change if the external field B changes, or the area of the loop changes, or the loop's orientation (θ) changes. Consider the example at the right.	<u>Example</u>: The loop area is 4.0 m^2, and doesn't change. The loop's orientation changes from 30^o to 60^o while the field strength changes from 3.0 T to 4.5 T. What is the flux change? $\Phi = 4.5\ (4.0) \cos 60^o$ $= 9.00$ T-m^2 $\Phi_o = 3.0\ (4.0) \cos 30^o$ $= 10.39$ T-m^2 $\Delta\Phi = \Phi - \Phi_o$ $= 9.00 - 10.39$ $= -1.39$ T-m^2

Faraday's Law

In what follows we will in general deal not with just *one* loop, but several joined together in a "coil" of N loops, or "turns," such as illustrated below:

Coil of wire:

N = number of turns

When the flux through a coil changes over time, an EMF (voltage) is "induced" in the coil, and the coil acts just as if it were a battery. The average EMF induced in the coil during the period of time t that the flux is changing is given by "Faraday's Law."

$$\varepsilon = N\,|\Delta\Phi|\,/t$$

If flux is not changing, the induced EMF is zero.

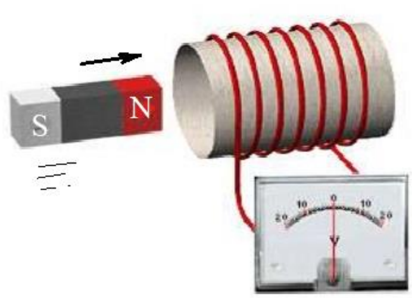
<u>Example:</u>

Over a three-millisecond time period, the flux through a coil of 20 turns changes by -4.0×10^{-4} T-m^2.

What average EMF was induced in the coil?

$\varepsilon = N\,|\Delta\Phi|\,/t$
$= 20\,(4.0 \times 10^{-4})/0.003$
$= 2.67$ V

Example:



A bar magnet is moving toward a wire coil of radius r = 0.08 m having seven turns, causing the magnetic field through the coil to change from 100 G to 250 G in 0.20 s.

What average EMF (in milli-volts, mV) is induced in the coil during this time period?

Assume the B vectors fluxing through the coil are perpendicular to the faces of the turns:
$\theta = 0^o$

$A = \pi\,(0.08)^2$
$\quad = 0.020\ m^2$

$\Delta\Phi = \Phi - \Phi_o$
$\quad = BA \cos 0 - B_oA \cos 0$
$\quad = (B - B_o)A$
$\quad = (250 \times 10^{-4} - 100 \times 10^{-4})\,(0.20)$
$\quad = 0.003\ T\text{-}m^2$

Calculate ε:

$\varepsilon = N\,|\Delta\Phi|\,/t$
$\quad = 7(0.003)/0.20$
$\quad = 0.105\ V$
$\quad = 105\ mV$

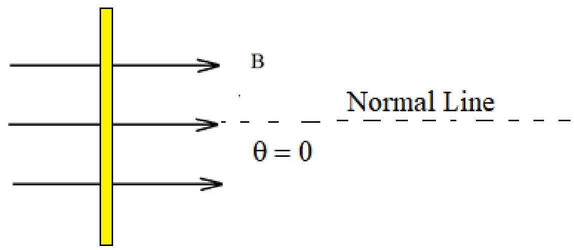
Example:

As shown below, a coil of 1000 turns and area 0.02 m^2 is perpendicular to a magnetic field whose intensity is 0.40 T.

In 0.001 seconds the coil is rotated so that its normal is perpendicular to the field. What average EMF is induced in the coil?

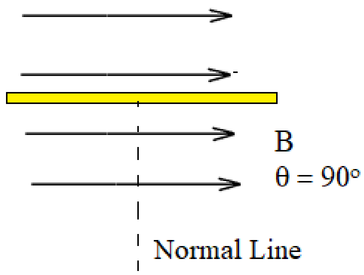
Edge View of Circular Coil Before

t = 0.000 s



Edge View of Circular Coil After

t = 0.001 s



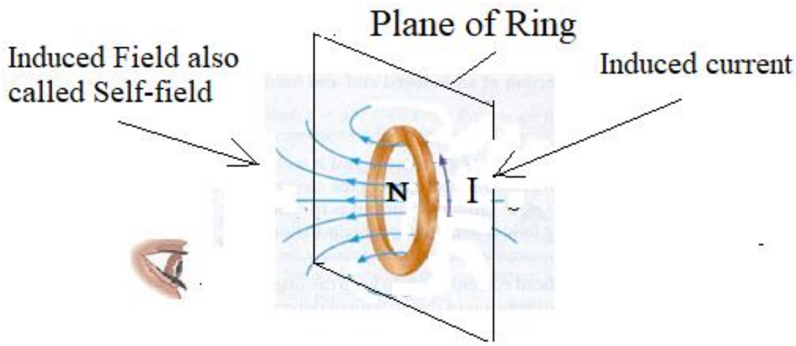
$$
\begin{aligned}
\Delta\Phi &= \Phi - \Phi_o \\
&= AB\cos\theta - A_oB_o\cos\theta_o \\
&= 0.02\,(0.40)\cos 90 - 0.02\,(0.40)\cos 0 \\
&= -0.008\ \text{T-m}^2
\end{aligned}
$$

$$
\begin{aligned}
\varepsilon &= N\,|\Delta\Phi|\,/t \\
&= 1000\,|(-0.008)|\,/0.001 \\
&= 8000\ \text{volts}
\end{aligned}
$$

Self-Flux

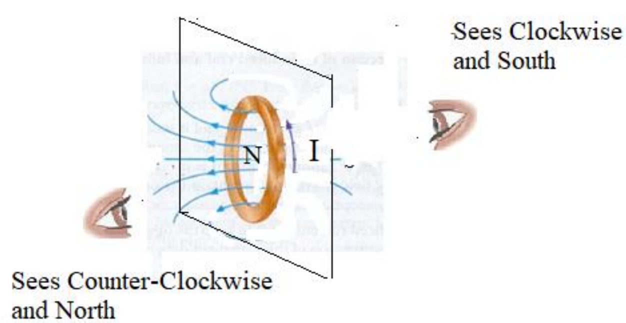
Faraday's Law tells us that a changing external magnetic flux through a coil, such as might be caused by a magnet moving near the coil, induces a current in the coil.

That current creates a second field, called the "self-field," which makes the coil an equivalent bar magnet, with a north face, and a south face.



The changing magnetic field that is causing this self-fileld is not shown.

Clockwise vs Counter-Clockwise Currents



Mnemonic: The words, "counter," and "north," go together. They share one letter in common: "n."

Lenz’s Law

We saw above that a moving magnet’s field can create a second field—a self-field—in a nearby coil.

Here is Lenz’s Law:

The change in external flux through a ring or coil is opposed by the coil’s self-flux.

Note: the change is only opposed, not completely negated.

More generally,

The effect opposes the cause.

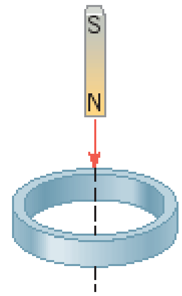
The effect is only to partially cancel the effect, not eliminate it entirely.

Example A:

As viewed by an observer moving with the falling magnet in the figure at the right, what is the direction of the induced current in the ring?

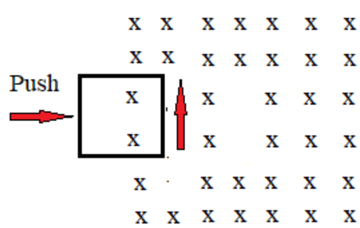
Cause: Magnet pulled Downward by
Effect: Magnet Pushed Upward by Ring

The current must be counter-clockwise in the ring in order that the top of the ring's equivalent bar magnet be north to slow the fall of the north end of the falling magnet.



Example B:

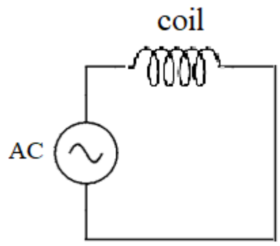
The square ring in the figure below is being pushed into a magnetic field. The flux through the ring is changing, so a current is induced in it. What is the direction of that current ?



Cause: Increased flux external flux
Effect: Opposing self-flux

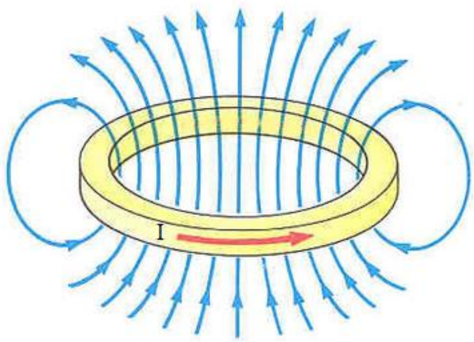
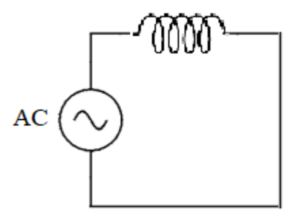
As the ring is pushed into the B-field, the inward external flux into the plane increases. To counteract the increasing inward flux, an <u>outward</u> induced (self) flux (not shown) out of the plane, caused by a counter-clockwise current ring current is needed.

Alternating Current (AC) Power Supplies



Unlike DC (direct current) batteries which supply current in one direction, either clockwise or counter-clockwise around a circuit, alternating current (AC) flows in alternating directions—clockwise, then counter-clockwise, and so on.

Self-Flux Caused by AC Current

Recall from Chapter 21 that a wire ring carrying current creates a “self-field” which is fluxing through ring. Such a “self-flux” is illustrated in the figure at the right.	 Self Flux
If a coil is connected to an AC power supply, the constantly changing current causes a constantly changing self-flux and voltage in each turn. The coil then acts like a set of AC batteries connected in series. The example below shows how to determine the voltage of each one of those turns.	

Example:

A coil of six turns is connected to an AC power supply whose voltage is 120 volts.

What is the AC voltage V induced in each turn?

Each turn experiences the same change in flux over the same time period, so the induced voltage is the same in every turn. By Lenz's Law, the effects (the induced voltages) oppose the cause (the AC power supply voltage): when one is positive (clockwise), the other is negative (counter-clockwise).

Let V symbolize the voltage induced in each of the six turns:

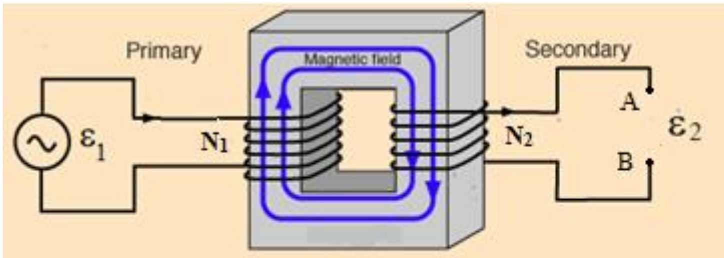
By Kirchoff's Law, the sum of the voltages is zero:

$$120 - 6V = 0$$
$$V = 20 \text{ volts}$$

The induced voltage per turn is 20 volts.

In general, if there are N turns, and the AC power supply voltage is ε, then $V = \varepsilon/N$.

Transformers Part One: The Primary



Turns of wire from an AC power supply wrap around the left side (the "primary") of an "iron donut."

The changing EMF from the AC power supply causes a changing current in each of the N_1 turns of the primary coil. In each turn, the changing current causes a changing self-flux and a self-induced voltage, V.

By the loop rule, the sum of the voltages around the primary circuit equals zero:

$\varepsilon_1 - N_1 V = 0$

Solve for V:

$V = \varepsilon_1 / N_1$

Transformers Part Two: The Secondary

In a well-designed transformer, whatever flux flows through each of the primary turns, also flows through each of the turns on the other side--the "secondary" side. Thus, the voltage V per turn is the same for all of the turns in the primary *and* secondary.

Using the Loop Rule for the secondary circuit:

$\varepsilon_2 = N_2 V$

$\quad = N_2 (\varepsilon_1 / N_1)$

$\quad = (N_2 / N_1)\, \varepsilon_1$

Re-written, we have "The Transformer Equation."

$$\varepsilon_2 = (N_2/N_1)\, \varepsilon_1$$

Transformer Applications

Here, again, is the transformer equation:

$$\varepsilon_2 = (N_2/N_1)\,\varepsilon_1$$

If N_2 is greater than N_1, then ε_2 is higher than ε_1. Such a transformer is called a "step up transformer."

If N_2 is less than N_1, then ε_2 is lower than ε_1. Such a transformer is called a "step down" transformer."

Note: While the output voltage from a step-up transformer is higher than the input voltage, the conservation of energy law tells us that *the output power can never be greater than the input power*. At most, the output power would equal the input power.

Also note: Some "stun" guns ("tasers") step up the voltage from a 3.0-volt battery by a factor of 1000, to 3,000 volts, which is sufficient to paralyze the large muscles of a mis-behaving human.

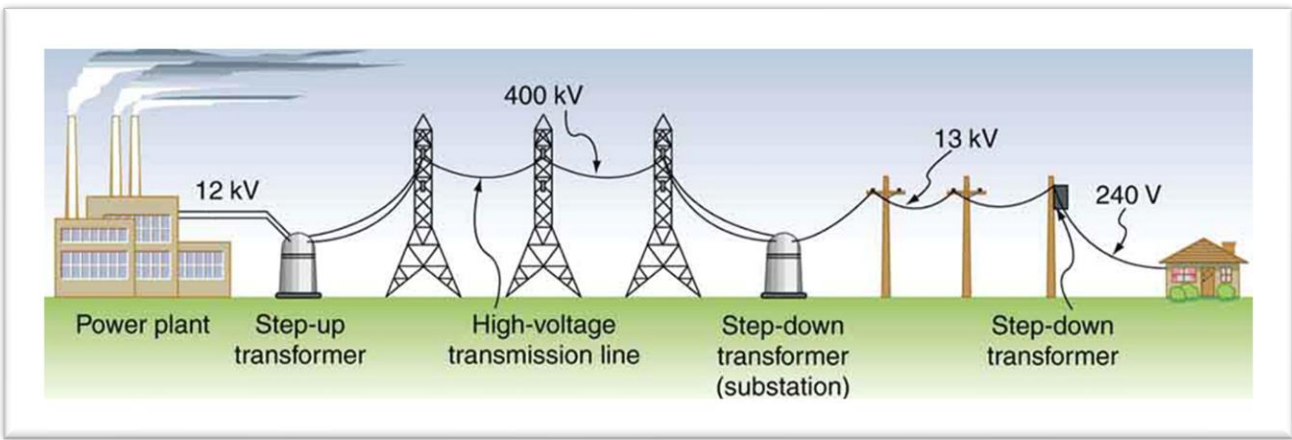
TASER M26C

If N_2 is less than N_1, the secondary EMF is lower than the primary EMF, and the transformer is called a "step down transformer."

Many mobile electronic devices have batteries that are charged at a voltage between 3.0 and 9.0 volts using a step-down transformer connected to a 120-volt AC wall outlet.

A step-down transformer on the telephone pole reduces power-plant sub-station voltage from 20,000 volts to 220 volts.

Transmission Line Power Loss



Electricity is often generated dozens of miles from where it is used. Although the resistance of a short length of power line is relatively low, over a long distance the resistance is substantial.

A power line of resistance R carrying current I loses energy at the rate $P = I^2R$. By minimizing the output current, the I^2R power losses can be reduced to acceptable levels.

The example below addresses the question of power line losses.

Example:

The resistance per mile of typical power lines is very small--about 0.40 Ω per mile. (Compare this to the 200-ohm resistance of a 60-watt incandescent light bulb filament.)

Consider a power plant outputting 10 megawatts (MW) of electrical power, destined for a city 20 miles away.

$$P = I\varepsilon$$
$$= 1.0 \times 10^7 \text{ W}$$

This power can be created in an uncountable number of ways; the only requirement is that the product of the output current I and EMF ε be 1.0×10^7 W.

Suppose the output current and voltage are 1.0×10^3 A, and 1.0×10^4 V. What would be the percentage power loss due to I^2R heating?

$R = (0.40\ \Omega/\text{mi})\ (20\ \text{mi})$
$= 8.0\ \Omega$

$P = I^2R$
$= 1000^2\ (8.0)$
$= 8.0 \times 10^6$
$= 8.0$ MW

Percentage Lost = (8.0/10.0) 100%
= 80%

Another Power Loss Example

For the power plant in the example above, what would have to be the output voltage and current in order that the loss be only 5% ?

$$
\begin{aligned}
P &= 0.05\ (10.0) \\
&= 0.50\ \text{MW} \\
&= 5.0 \times 10^5\ \text{W}
\end{aligned}
$$

$$I^2\ (8.0) = 5.0 \times 10^5$$

$$I = 250\ \text{A}$$

$$\varepsilon(250) = 1.0 \times 10^7$$

$$\varepsilon = 40{,}000\ \text{V}$$

A series of step-down transformers at sub-stations close to the city provides the 110-V and 220-V voltages needed by electrical devices in the home and factory.