Magnetism and Magnetic Forces

Like poles repel, unlike poles attract.

Magnets have two “poles, one “north,” the other “south.” Bar magnets are “magnetic dipoles.”

Magnetic dipole fields look just like the electric field of electric dipoles.

Magnetic field lines flow away from the north pole and arrive at the south pole.

A compass needle is a miniature bar magnet, one end of which is “north” (the “red” end in the figure below) and the other end of which is “south.”
Earth has a magnetic field that may be imagined to be due to an 8,000 mile-long bar magnet, as suggested by the figure below.

The north pole of Earth’s equivalent bar magnet is near Earth’s geographic south pole, and vice-versa.

The “magnetic field intensity” at any point in a magnetic field is symbolized as $B$.

The units of $B$ is “tesla” (T).

$10,000 \text{ gauss (G)} = 1 \text{ T}$

Note: near the equator the magnetic field arrows are roughly parallel to the ground, and point north. Near Earth’s equator: $B = 0.5 \text{ G}$

We often say, “the $B$-field” when we refer to the direction of the magnetic field, as well as when we speak of the value of the magnetic field intensity. For example, the $B$-field near Earth’s equator is about 0.5 gauss.

<table>
<thead>
<tr>
<th>North geographic pole, South magnetic pole</th>
<th>South geographic pole, North magnetic Pole</th>
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The north pole of the compass needle is attracted to Earth’s south magnetic pole, which is near Prince Edward Island, Canada.

A compass needle lines up parallel to magnetic field lines. The red end of the compass bar-magnet needle is its north pole, and points to Earth’s south magnetic pole, which is near the north geographic pole.
Magnetic Force on Moving Charges: The Right-Hand Rule

Use the “right-hand rule” to determine the direction of the force on a positive moving charge:

- Flatten right hand with fingers pointing in the direction of the magnetic field arrows.
- Twist hand until thumb points in the direction of the velocity vector.
- Palm faces direction of force: Think, “palm-push.”
- If the charge is negative, the back of the hand faces the force direction.

To determine the value of the magnetic force, use the equation below:

\[ F = QvB \sin \theta \]

As shown above, \( \theta \) is the angle between the thumb and the fingers, i.e., the angle between the velocity vector and the magnetic field vector.

Maximum force occurs when \( \sin \theta \) is maximum, at \( \theta = 90^\circ \). Charged particles moving parallel to magnetic field lines (\( \theta = 0^\circ \)) experience zero magnetic force, because \( \sin 0^\circ = 0 \).

**Example:**

The magnetic field in a laboratory points from an observer’s right to her left, parallel to the floor, and has a uniform value of 3.0 T everywhere. A proton is fired in front of the observer from the floor upward at speed 2.0 x 10^6 m/s

(a) What is the magnetic force on the proton?

\[
F = QvB \sin \theta \\
= (1.6 \times 10^{-19}) (2.0 \times 10^6) (3.0) \sin 90^\circ \\
= 9.6 \times 10^{-13} \text{ N}
\]

(b) What is the direction of the force?

Answer: Initially, toward the observer.
Example A:

Suppose a magnetic field created in a laboratory points from floor to ceiling, and an electron is fired in front of an observer in the lab, parallel to the ground, from your left to your right. In which direction will the electron be deflected?

Answer: Away from the observer.

(Note: the back of the right hand is used because the charge is negative.)

Example B:

A proton is fired upward from the equator. In which direction will it be deflected?

Answer: westward

Example C:

A proton at the equator is fired northward. In which direction will it be deflected?

Answer: Recall, at the equator the B-field is parallel to the ground, and points north. Charges moving parallel to magnetic field lines experience zero magnetic force, because $\theta = 0$.

Charges fired south would likewise experience zero force, because $\theta = 180^\circ$ and $\sin 180^\circ = 0$. 
As shown in the figure above, a magnetic field in a laboratory points from ceiling to floor. Looking down from the ceiling, an observer sees the “tail feathers” of the magnetic field arrows (“x’s”).

At Point P, a proton is fired from left to right, parallel to the floor.

By the right-hand rule, the magnetic force is perpendicular to the direction of motion, always pointing to the center of the circular path around which the proton travels. This force is analogous to the tension in a string at the end of which is an object traveling in a circular path. The tension force is always perpendicular to the velocity vector and always points to the center of the circle.

Derive an expression for the radius of the circular path around which the proton travels.

\[ F = ma \]
\[ evB \sin \theta = \frac{mv^2}{r} \]

The angle \( \theta \) between the velocity vector and the magnetic field arrows is 90\(^\circ\), so \( \sin \theta = 1 \):

\[ evB \sin 90 = \frac{mv^2}{r} \]
\[ r = \frac{mv}{eB} \]

**Example:**

An electron and a proton, each having the same kinetic energy, \( K \), enter a magnetic field. What is the ratio of the orbital radii of the circular paths followed by the two objects? Note: the proton’s mass is about 2000 times the electron’s mass.

First, note that

\[ K = \frac{1}{2} mv^2 \]
\[ = \frac{(mv)^2}{2m} \]

Solve the equation above for the momentum, \( mv \):

\[ (mv) = (2mK)^{1/2} \]

Using the result at bottom left:

\[ r = \frac{(mv)}{eB} \]
\[ = \frac{(2mK)^{1/2}}{eB} \]

\[ \frac{r_1}{r_2} = \frac{(2m_1K)^{1/2}/eB}{(2m_2K)^{1/2}/eB} \]
\[ = \left( \frac{m_1}{m_2} \right)^{1/2} \]
\[ = (2000)^{1/2} \]
\[ = 44.72 \]

The orbital radius of the proton is 44.72 times greater than the orbital radius of the electron.
Magnetic Field Due to a Long Straight Wire

A current-carrying straight wire is said to be “long” if its length is much greater than the distances of interest from the wire. The magnetic field curves surrounding the wire are circles.

The direction of the magnetic field arrows is determined by applying “the second right-hand rule”:

Hold the wire in your right hand with your thumb pointing in the direction of the conventional current. Imagine your fingernails are the arrowheads on the circles. As viewed by the observer below, the magnetic field lines circulate counterclockwise.

Below: View from the Side of the Wire

Dots are tips of B arrows coming out of the plane.

Crosses above are the “tail feathers” of B arrows entering the plane.
Magnetic Field Intensity Due to a Long Straight Wire

The current $I$ in the diagram below is out of the plane.

Magnetic Constant, $\mu_0$:

$\mu_0 = 4\pi \times 10^{-7}$ T-m/A

$B = \frac{\mu_0 I}{2\pi r}$

Example A:

How close (in mm) to a long straight wire carrying 0.8 A is the magnetic field intensity 0.5 G, the same as Earth’s near the ground at the equator?

$B = \frac{\mu_0 I}{2\pi r}$

$0.5 \times 10^{-4} = \frac{(4\pi \times 10^{-7}) (0.8)}{2\pi r}$

$r = 0.0032$ m

$= 3.2$ mm

Example B:

The magnetic field intensity at a certain distance from a long, straight, current-carrying wire is 270 G. What is the value of $B$ one-third as far from the wire?

Answer:

If $r$ is reduced to one-third, then the denominator of the $B = \frac{\mu_0 I}{2\pi r}$ equation is reduced to one-third of its previous value. Thus, the ratio is increased to three times its previous value:

$B = 3 (270)$

$= 810$ G
Example:

The figure below shows four long, straight, parallel, current-carrying wires each carrying the same current $I$ perpendicular to the plane. Currents 1 and 2 are into the plane, while Currents 3 and 4 are out of the plane.

What is the magnetic field at point $P$ equidistant from each of the four wires?

Answer is provided at the right.

Solution:

The length of the diagonal is $a\sqrt{2}$, so the distance $r$ from a wire to Point $P$ is $\frac{1}{2} a\sqrt{2}$:

$$r = \frac{1}{2} a\sqrt{2}$$

The contribution to the total $B$ field at Point $P$ is the sum of the four fields, each one being the same as any other:

$$B_n = \frac{\mu_0 I}{2\pi r}, \ n = 1, 2, 3, 4$$

$$= \frac{\mu_0 I}{2\pi(\frac{1}{2} a\sqrt{2})}$$

$$= \frac{\mu_0 I}{\pi a\sqrt{2}}$$

The $x$-components of the four fields add, while the $y$-components cancel. The four $x$-components are all the same, so the sum of them equals four times the $x$-component of any one of them:

$$B_x = B_{1x} + B_{2x} + B_{3x} + B_{4x}$$

$$= 4 B_{1x}$$

$$= 4 B_1 \cos 45$$

$$= 4 B_1 \frac{1}{2}\sqrt{2}$$

$$= 4 \frac{\mu_0 I}{(\pi a\sqrt{2})} \frac{1}{2}\sqrt{2}$$

$$= 2\mu_0 I /\pi a$$

Total magnetic field points to the right.
Example:  At the right are shown two long parallel straight wires, separated by a distance 2d, each carrying current I in the same direction.

(a) What is $B$ midway between the wires, at Point P?

(b) What is $B$ at Point Q, a distance d from Wire 2?

Arbitrarily choose “into the plane” to be the positive direction, and “out of the plane” to be the negative direction.

(a) $B_1 = \mu_0 I/2\pi d$ (into plane)
   $B_2 = -\mu_0 I/2\pi d$ (out of plane)

   $$B = B_1 + B_2 = \frac{\mu_0 I}{2\pi d} - \frac{\mu_0 I}{2\pi d} = 0$$

(b) $B_1 = \mu_0 I/[2\pi(3d)]$ (into plane)
   $B_2 = \mu_0 I/[2\pi d]$ (into plane)

   $$B = B_1 + B_2 = \frac{\mu_0 I}{2\pi(3d)} + \frac{\mu_0 I}{2\pi d} = [\frac{\mu_0 I}{2\pi d}] (1/3 + 1) = [\frac{\mu_0 I}{2\pi d}] (4/3) = (2/3) \frac{\mu_0 I}{\pi d}$$
Current Rings and the Third Right Hand Rule

With the thumb of the right hand perpendicular to the fingers, grab the axis of the ring with the fingers curling in the same direction as the current.

The thumb shows the direction of $B$ at the center of the ring.

Current-Carrying Ring: Equivalent to a Bar Magnet

The magnetic field of a current ring is identical to a bar magnet’s.
B at the Center of a Circular Ring

For circular rings, the value of B at the center is shown below:

\[ B = \frac{\mu_0 I}{2R} \]

The B field at the center of the ring above points out of the plane.

Simple equations for B at points other than at the center do not exist.

Example A:

What is the current in a circular ring of radius 0.10 m that will cause a magnetic field intensity \( B = 0.003 \) T at the center?

\[ B = \frac{\mu_0 I}{2R} \]

Re-write as:

\[ I = 2RB/\mu_0 \]

\[ = 2(0.10)(0.003) / (4\pi \times 10^{-7}) \]

\[ = 477.46 \text{ A} \]

Example B:

A circular ring of radius \( R = 4.0 \) cm is twisted into a long straight wire carrying current \( I = 2.5 \) A. What is the B field at the center of the ring, in gauss?

B at the center is the sum of two contributions: one due to the straight wire, and one more from the ring. Both contributions point out of the plane according to the right-hand rules for rings and straight wires. Letting “out of the plane” this time be the positive direction, the two contributions are therefore both positive. Their sum is shown below:

\[ B = \frac{\mu_0 I}{2\pi R} + \frac{\mu_0 I}{2R} \]

\[ = 5.18 \times 10^{-5} \text{ T} \]

\[ = 0.518 \times 10^{-4} \text{ T} \]

\[ = 0.518 \text{ G} \]
Magnetic Force on a Straight Current-Carrying Wire

The figure below shows how the right hand is used to determine the direction of the magnetic force on a length L of straight wire carrying current I.

Using a flat right hand, make the fingers point along the direction of the magnetic field B.

Twist the hand until the thumb is aligned with the current-carrying wire (thumb shown below; wire is not shown), and pointing in the direction of the conventional current I. The angle between the current vector and the B vector is \( \theta \).

\[ F = ILB \sin \theta \]

The palm faces in the direction of the force F on the wire segment. These are the same steps followed when the direction of the magnetic force on a moving charge is determined--the first right-hand rule.
Example:

Long straight current-carrying parallel wires lie in the same plane and are separated by a distance \( r \).

Consider a segment of length \( L \) along the top wire:

What is the magnetic force \( F \) on that segment?

\[
F = I_1 LB \sin \theta
\]

The upper wire segment is a distance \( r \) from the lower wire, which produces at the upper location the B-field shown below:

\[
B = \frac{\mu_0 I_2}{2\pi r}
\]

By the 2\textsuperscript{nd} Right-Hand Rule, the direction of \( B \) is perpendicular to the top wire, so \( \sin \theta = 1 \).

Finally,

\[
F = I_1 LB = I_1 L \left( \frac{\mu_0 I_2}{2\pi r} \right) = \frac{\mu_0 I_1 I_2}{2\pi r}
\]

The first right-hand rule shows that the \( F \) vector points toward the lower wire.

Thus, the top wire is attracted to the lower one, and, by Newton’s 3\textsuperscript{rd} Law, the lower wire is likewise attracted to the top one. They each move toward the other.

Convince yourself that if the currents were in opposite directions the wires would repel each other.
Example:

Long parallel straight wires are 0.12 m apart and carry the same current in opposite directions. The magnetic force on a 3.0-m portion of one of the wires is 2.0 N.

What is the magnetic field strength (in gauss) midway between the wires?

Solution:

First, find the current, I:

\[ F = ILB \sin \theta \]
\[ 2.0 = I (3.0)B \sin 90 \]

Substitute \( B = \frac{\mu_0 I}{2\pi(0.12)} \):

\[ 2.0 = I(3.0) \frac{\mu_0 I}{2\pi(0.12)} \]
\[ I = 632.46 \text{ A} \]

Midway, each wire contributes the same B, and in the same direction:

\[ B = \frac{\mu_0 I}{2\pi d}, \text{ where } d = \frac{1}{2} r \]
\[ = \frac{\mu_0 432.46}{2\pi(0.06)} \]
\[ = 0.0021 \text{ T} \]

Total:

\[ B = 0.0021 + 0.0021 \]
\[ = 0.0042 \text{ T} \]
\[ = 42 \text{ G} \]