Physics $25 \quad$ Chapter 21
Magnetism and Magnetic Forces
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Attraction


A compass needle is a miniature bar magnet, one end of which is "north," and the other end of which is "south."


## Magnetic Field Lines



Magnetic field lines point away from the north pole, and toward the south pole.

If a compass needle were placed anywhere on one of the magnetic field lines it would twist until the needle's north end is directed away from the bar magnet's north pole. The needle would then travel along the line toward the magnet's south pole, always tangent to that magnetic field line.

For example, if the compass needle were placed at Point A in the figure below, it would follow the path ABCDE .



## Earth's Magnetic Field

Earth's magnetic field arises from the circulation of iron and other metal ions deep within Earth's core. Earth field may be thought of as arising from a bar magnetic 8,000 miles long, with its south pole in Canada, and its north pole in Antarctica. Thus, Earth's north magnetic pole is located at Earth's south geographic pole, and vice-versa.

Earth's magnetic field arrows leave Antarctica, wrap around the Earth, and land near Prince Edward Island in Canada.

Note: as is always the case for magnetic field lines, the magnetic field lines that encompass Earth, leave the north magnetic pole (in antarctica) and land on the south magnetic pole (in Canada).

"Magnetic field intensity" is symbolized as B , and is in units of "tesla" (T).
$1.0 \mathrm{~T}=10,000$ gauss ( G )
Near the ground at the equator, $\mathrm{B}=1 / 2 \mathrm{G}$.

## Magnetic Force on Moving Charged Objects

Use the "right-hand rule" to determine the direction of the magnetic force on a moving charge.


1. With flattened right hand, point the thumb in the direction of the velocity vector.
2. Twist the flattened hand while preserving the direction of the velocity vector (the thumb) until the fingers point in the direction of the magnetic field vector.
3. If the charge is positive, the force vector is perpendicular to the palm, pointing away from the palm. The charge in the figure above is positive.
4. If the charge is negative, the force vector be perpendicular to the the back of the hand, pointing away from the back of the hand.

## Example A:

Suppose a magnetic field created in a laboratory points from floor to ceiling, and a proton (a positively-charged object) is fired parallel to the floor in front of you, from your left to your right.

In which direction initially will the electron be deflected?


The figure above shows your right hand with its fingers pointing toward the ceiling, and its thumb pointing to your right. The palm faces you, so the magnetic force on the proton points toward you.

## Example B:

What if the charged object in the example above were an electron? In which direction initially would it be deflected?

The electron is negatively charged: the force vector is perpendicular to the back of the hand, and points away from it-away from you.

| Example: |
| :--- |
| A proton is fired upward from |
| the equator. In which direction |
| will it be deflected? |
| By the right-hand-rule, the |
| force vector points westward. |
| If an electron had been fired |
| instead, the force vector would |
| point eastward. |

## Example:

The rectangular plane below represents a magnetic field that points into the plane, away from the reader.
(The x's shape suggests what the viewer would see if she were looking at the back of an archery arrow moving away from her.)


Four charges are fired into the field. The magnetic force directions are indicated in below:

1: downward
2: downward
3. to the right

4: to the right



The arrows above represents a magnetic field that points to the right.

Two charges are fired into the field. The magnetic force directions are indicated below:
1: Out of the plane, toward the reader.
2: Out of the plane, toward the reader.

## Magnetic Force on a Moving Charge

To determine the value of the magnetic force on a moving charge, use the equation below:

$$
\mathrm{F}=\mathrm{QvB} \sin \theta
$$



As shown above, $\theta$ is the angle between the velocity vector and the magnetic field vector.

Maximum force occurs when $\sin \theta$ is maximum, which occurs when $\theta=90^{\circ}$ and $\sin \theta=1.0$.

Charged particles moving parallel or anti-parallel to magnetic field lines experience zero magnetic force: If they're moving northward, for example, $\theta=0^{\circ}$ and $\sin 0=0$.

If they're moving southward, $\theta=180^{\circ}$, and $\sin 180^{\circ}=0$.

## Example:

The magnetic field in a laboratory points from an observer's right to her left, parallel to the floor, and has a uniform value of 3.0 T everywhere.

A proton $\left(\mathrm{q}=1.6 \times 10^{-19} \mathrm{C}\right)$ is fired in front of the observer from the floor upward at speed $2.0 \times 10^{6} \mathrm{~m} / \mathrm{s}$. The velocity vector makes an angle of $30^{\circ}$ relative to the normal line to the floor.
(a) What is the magnetic force on the proton?
$\mathrm{F}=\mathrm{QvB} \sin \theta$
$=\left(1.6 \times 10^{-19}\right)\left(2.0 \times 10^{6}\right)(3.0) \sin 30^{\circ}$
$=4.8 \times 10^{-13} \mathrm{~N}$
(b) What is the initial direction of the force, as seen by the observer?

Answer: Toward the observer.

## Example:

In a laboratory a magnetic field points from ceiling to floor. A proton is fired at Point P parallel to the floor. The figure below is what an observer looking down from the lab ceiling sees.


A proton (charge e) is fired at Point P parallel to the floor. By the right-handrule, the force on the proton is always perpendicular to its velocity vector:

As the direction of the velocity changes, the force's direction likewise changes, and that force always points to the center of the circle around which the proton orbits. This magnetic force pulling on the proton, causing it to travel in a circular path, is analogous to the pull by a string on an object swinging in a horizontal circular path.

Obtain an expression for the radius of the orbit in terms of the proton's momentum, $\mathrm{p}=\mathrm{mv}$.

$$
\begin{gathered}
\mathrm{F}=\mathrm{ma} \\
\mathrm{evB} \sin 90^{\circ}=\mathrm{mv}^{2} / \mathrm{r} \\
\mathrm{r}=\mathrm{mv} / \mathrm{eB}
\end{gathered}
$$

In terms of momentum $p=m v$, we have

$$
\mathrm{r}=\mathrm{p} / \mathrm{eB}
$$

## Magnetic Field Due to a Long Straight Wire

A current-carrying straight wire is said to be "long" if its length is much greater than the observer's distance from the wire.

Without proof, we declare that the magnetic field of the currentcarrying wire are curves surrounding the wire in concentric circles of ever-widening radii, out to infinity. The direction of the magnetic field arrows attached to the circles is determined by applying the rule described below.

The Second Right Hand Rule


Hold the wire in your right hand with your thumb pointing in the direction of the conventional current. Your fingers curling around the wire mimic the curl of the magnetic field curves around the wire. Imagine that the fingernails are the magnetic field arrowheads.

Concentric circular B-field curves exists in every imagined plane intersecting the wire; three such sets of curves are shown below

Below are shown three concentric circles lying in three planes; the planes, not shown, are perpendicular to the wire.


## Magnetic Field Due to a Long Straight Wire



## Magnetic Field Intensity Due to a Long Straight Wire

| The current I in the diagram below is out of the plane, toward the reader. <br> Magnetic Constant, $\mu_{o}$ $\begin{aligned} & \mu_{0}=4 \pi \times 10^{-7} \mathrm{~T}-\mathrm{m} / \mathrm{A} \\ & \mathrm{~B}=\frac{\mu_{0} \mathrm{I}}{2 \pi \mathrm{r}} \end{aligned}$ | Example A: <br> How close (in mm ) to a long straight wire carrying 0.8 A is the magnetic field intensity 0.5 G, the same as Earth's field near the ground at the equator? $\begin{aligned} & \mathrm{B}=\mu_{0} \mathrm{I} / 2 \pi \mathrm{r} \\ & 0.5 \times 10^{-4}=\left(4 \pi \times 10^{-7}\right)(0.8) / 2 \pi \mathrm{r} \\ & \mathrm{r}=0.0032 \mathrm{~m} \\ &=3.2 \mathrm{~mm} \end{aligned}$ | Example B: <br> The magnetic field intensity at a certain distance from a long, straight, current-carrying wire is 270 G . What is the value of B one-third as far from the wire? $\begin{aligned} \mathrm{B} & =\mu_{\mathrm{o}} \mathrm{I} /(2 \pi \mathrm{r}) \\ & =270 \mathrm{G} \end{aligned}$ <br> New r is $\mathrm{r} / 3$ : $\begin{aligned} \text { New } \mathrm{B} & =\mu_{\mathrm{o}} \mathrm{I} /(2 \pi \mathrm{r} / 3) \\ & =3 \mu_{\mathrm{o}} \mathrm{I} /(2 \pi \mathrm{r}) \\ & =3(270) \\ & =810 \mathrm{G} \end{aligned}$ |
| :---: | :---: | :---: |


| Example: |
| :--- | :--- |
| At the right are shown two long parallel straight |
| wires, separated by a distance 4.0 cm , each |
| carrying a 6.0 A . |
| (a) What is B midway between the wires--at |
| Point $\mathrm{P}, 2.0 \mathrm{~cm}$ from each wire? |
| The field intensity due to the left wire points |
| into the plane, while the identical field intensity |
| from the right wire points out of the plane. The |
| two different contributions cancel, so the total |
| field intensity at Point P is therefore zero. |
| (b) What is B at Point Q, a distance 2.0 cm |
| from Wire 2, but 6 cm from Wire 1 ? |
| Arbitrarily choose "into the plane" to be the |
| positive direction, and "out of the plane" to be |
| the negative direction. |
| By the second right-hand rule, the two fields at |
| Point Q point into the plane, and therefore add: |
| $\mathrm{r}_{1}=6.0 \times 10^{-2} \mathrm{~m}$ |
| $\mathrm{r}_{2}=2.0 \times 10^{-2} \mathrm{~m}$ |
| $\mathrm{~B}=\mu_{\mathrm{o}}(6.0) / 2 \pi \mathrm{r}_{1}+\mu_{\mathrm{o}}(6.0) / 2 \pi \mathrm{r}_{2}$ |
| $=8.00 \times 10^{-5} \mathrm{~T}$ |

## Current Rings and the Third Right Hand Rule

To determine the general direction of
the B-field vector at the center of a
current ring, imagine grasping the
current ring as shown in the figure,
with your right hand with your finger
nails pointing in the same direction as
the circulating current. The thumb
then points in the direction of the
magnetic field vector on the ring's axis.
Remember: magnetic field arrows
leave the north pole of a magnet, so the
hand in the figure is above the north
pole of the equivalent bar magnet.


## Clockwise vs Counter-Clockwise Currents



## B at the Center of a Circular Ring

In the discussion above, we learned how to determine the direction of a magnetic field on the axis of a current ring, at the center of the ring. In this section, we learn how to determine the value of the magnetic field intensity, B , at the center of the ring.

For circular rings, the value of $B$ at the center is shown below:


The B field vector at the center of the ring above points out of the plane, according to the third right-hand rule.

Simple equations for B at points other than at the center do not exist.

## Example:

What is the current in a circular ring of radius 0.10 m that will cause a magnetic field intensity $\mathrm{B}=0.003 \mathrm{~T}$ at the center?

$$
\begin{aligned}
\mathrm{B} & =\mu_{\mathrm{o}} \mathrm{I} / 2 \mathrm{R} \\
0.003 & =\left(4 \pi \times 10^{-7}\right) \mathrm{I} /(2 \times 0.10) \\
\mathrm{I} & =477.46 \mathrm{~A}
\end{aligned}
$$

## Example:



A long straight wire carrying 2.5 A current is twisted as shown above, creating a circular ring of radius $\mathrm{R}=4.0 \times 10^{-2} \mathrm{~m}$. What is the value of B at the center of the ring, in gauss?
$B$ at the center is the sum of two contributions: one due to the straight wire, and one more from the ring. Both contributions point out of the plane according to the right-hand rules for rings and straight wires.

$$
\begin{aligned}
\mathrm{B} & =\mu_{\mathrm{o}} \mathrm{I} / 2 \pi \mathrm{R}+\mu_{\mathrm{o}} \mathrm{I} / 2 \mathrm{R} \\
& =5.18 \times 10^{-5} \mathrm{~T} \\
& =0.518 \times 10^{-4} \mathrm{~T} \\
& =0.518 \mathrm{G}
\end{aligned}
$$

## Magnetic Force on Current-Carrying Wires

The direction of the magnetic force on a stream of charged particles is determined in exactly the same way as the direction of the magnetic force on a single moving charge.


The direction of the magnetic force on a wire along which a multitude of charges are moving (a current is flowing) is found using the same right-hand rule that is used for a single moving charge:


The fingers of the flattened right hand point in the direction of the magnetic field lines, the thumb points in the direction of the conventional current; the force vector is perpendicular to the palm.

## Examples of Right-Hand Rule for Current-Carrying Wires

(It's the same rule used for single charges.)

| x x x x x x x x x x x x x x x x x x x | X XXXXXXXXXXXXXXXXXXX |
| :---: | :---: |
|  | x x XXXXXXXXXXXXXXXXXXX |
|  | $\underset{\mathrm{I}}{\mathrm{x} \times \mathrm{x} \quad \mathrm{xxxxxxxxxxxxxx}}$ |
|  |  |
|  |  |
|  | x x XXXXXXXXXXXXXXXXXXX |
| Field is into the plane. | Field is into the plane. |



## Example:

What is the force the long wire exerts on the 3.0-meter wire?
Note: The field enveloping the 3.0-meter wire is the field due the long, 400 A wire.


## Calculate B:

$\mu_{0}=4 \pi \times 10^{-7} \mathrm{~T}-\mathrm{m} / \mathrm{A}$
$\mathrm{I}=400 \mathrm{~A}$
$\mathrm{r}=0.02 \mathrm{~m}$
$\mathrm{B}=\mu_{\mathrm{o}} \mathrm{I} / 2 \pi \mathrm{r}$

$$
=4.00 \times 10^{-3} \mathrm{~T}
$$

## Calculate F:

$$
\begin{aligned}
\mathrm{I} & =200 \mathrm{~A} \\
\mathrm{~L} & =3.0 \mathrm{~m} \\
\mathrm{~F} & =\mathrm{ILB} \sin \theta \\
& =(200)(3.0)\left(4.0 \times 10^{-3}\right) \sin 90 \\
& =2.4 \mathrm{~N}
\end{aligned}
$$

The right-hand rule shows that the force on the upper wire is downward. What is the force and direction exerted on the bottom wire by the top wire, according to Newton's Third Law?

Answer: 2.4 N , upward.

## Magnetic Force between Parallel Current-Carrying Wires



