Physics 25 Chapters 19-20 Electric Circuits

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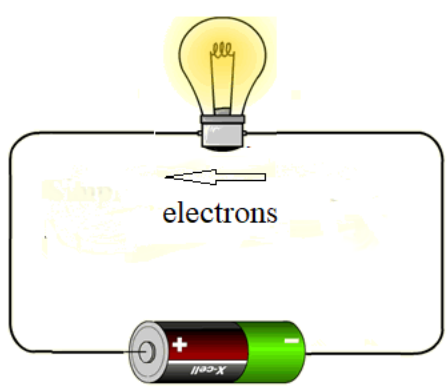
Batteries store chemical energy that's convertible into electrical energy. A new nine-volt battery stores about 20,000 joules of chemical energy. A new car battery stores about two million joules.

In the figure below, which constitutes what we call an electrical "circuit," electrons produced from chemical reactions in the battery leave the negative terminal of the battery and circulate around the circuit to the positive terminal.



Battery Current

Example:

In 10.0 seconds, 2×10^{20} electrons leave the battery's negative terminal and arrive at its positive terminal.

(a) What is the absolute value of this quantity Q of charge?

The absolute value of the electron charge is e.

$$Q = (2 \times 10^{20})\,(1.6 \times 10^{-19})\ \text{C}$$
$$= 32\ \text{C}$$

The battery current is defined to be the quantity of charge Q leaving the battery each second:

$$I = Q/t$$

(b) What was the current from this battery during the 10-second period?

$$I = 32\ \text{C}/\ 10\ \text{s}$$
$$= 3.2\ \text{C/s}$$

Let the "ampere" (A) be the SI unit of current:

$$1.0\ \text{ampere (A)} = 1.0\ \text{C/s}$$

(c) What is the current in SI units?

$$I = 3.2\ \text{A}$$

Electromotive Force (EMF)

Batteries are rated according to their "electromotive force" (EMF).

Symbol: ε

EMF is not a force; it's the energy the battery gives per coulomb of charge that leaves the battery. The units of EMF, therefore, are joules per coulomb (J/C).

Let 1.0 "volt" = 1.0 J/C

The "volt" is the standard international unit for voltage. Battery EMF is usually just called "battery voltage," or "voltage."

For example, suppose a battery provides 1000 J of energy to 50 C of charge leaving the battery.

What is the battery's voltage?

$$\begin{aligned}\varepsilon &= 1000\text{ J}/50\text{ C}\\ &= 20\text{ J/C}\\ &= 20\text{ volts}\end{aligned}$$

Note the temptation to abbreviate "volt" as "V" is strong, but we will use "volt" because later in this chapter we will use "V" as the symbol for another circuit quantity.

Example:

8,000 joules of chemical energy are stored in a 16-volt battery.

(a) How many coulombs of charge will exit the battery before it's dead?

The battery supplies 16 J of energy to each coulomb leaving the battery:

$$8{,}000 \text{ J} / (16 \text{ J/C}) = 500 \text{ C}$$

(b) Suppose it took 20 seconds for the 500 C charge to leave the battery. What was the battery current during that time?

$$\begin{aligned} I &= Q/t \\ &= (500 \text{ C}) / (20 \text{ s}) \\ &= 25 \text{ C/s} \\ &= 25 \text{ ampere (A)} \end{aligned}$$

(c) What was the battery's power output during that 20-second period?

$$\begin{aligned} P &= \text{Energy/Time} \\ &= (8{,}000 \text{ J}) / (20 \text{ s}) \\ &= 400 \text{ J/s} \\ &= 400 \text{ watts (W)} \end{aligned}$$

(d) Show that the "PIE" equation below can be used to find the output power of this battery:

$$P = I\varepsilon$$

$$\begin{aligned} P &= (25)\,(16) \\ &= 400 \text{ W} \end{aligned}$$

We will soon show how the equation $P = I\varepsilon$ is derived.

Derive Battery Power Equation

Energy = Votage x Charge

Energy/Time = Voltage x Charge/Time

$P = \varepsilon I$

$= I \varepsilon$

<u>Example A</u>:

What is the power output of a 40-volt battery outputting 6.0 amperes?

$P = I\varepsilon$

$= (6.0\ A)\ (40\ V)$

$= 240\ W$

<u>Example B</u>:

The energy stored in a new 9-volt battery is about 20,000 J. The current from the battery is 0.015 A.

After how many hours of use will this battery be drained of all of its energy (be "dead") ?

$P = I\varepsilon$

$= 0.015\ (9)$

$= 0.135$ watts

$= 0.135$ J/s

During each second of operation, 0.135 J of chemical energy are consumed.

(20,000 J)/(0.135 J/s) = 148,148 s

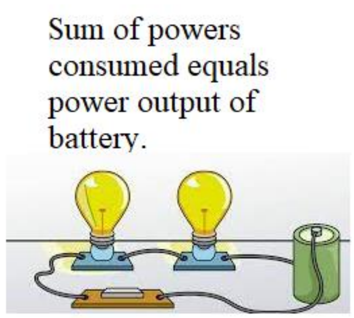
= 41 hours

Conservation of Energy in Circuits

Example:

The voltage of the battery in the circuit below is 2.0 volts, and the current output is 7.5 milli-amperes (mA). energy to two light bulbs. Two light bulbs consume the electrical energy provided by the battery. One of the bulbs consumes energy at the rate of 9.0 milli-watts (9.0 milli-joules each second). At what rate (in milli-watts) does the other bulb consume energy?

The rule that applies here is conservation of energy: the total amount of energy consumed by the light bulbs each second equals the energy provided to a circuit by the battery each second.



$$P = I\mathcal{E}$$
$$= (7.5 \times 10^{-3})(2.0)$$
$$= 15.0 \times 10^{-3} \text{ watts}$$
$$= 15.0 \text{ milli-watts (mW)}$$

One of the bulbs consumes 9.0 mW; the other bulb consumes the rest:

15.0 – 9.0 – 6.0 mW

Resistance

Objects (such as light bulbs) through which current pass have a property called "resistance," so-called because these "resistors" resist, impede, restrict, and limit the flow of electrons (current) from the battery.

The greater the resistance in a circuit, the smaller will be the current passing through it.

Symbol: R
Units: "ohms" (Ω).

The graphic symbol for a resistor reminds one of a series of speed bumps impeding the flow of traffic.

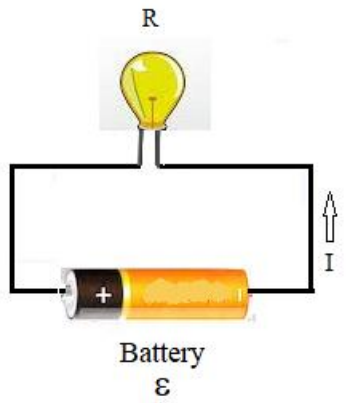
Resistor Symbol:

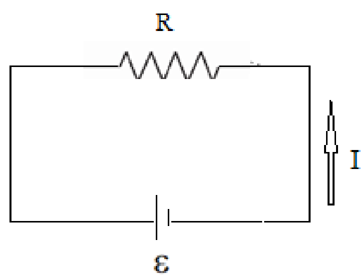
<u>Examples of Resistances</u>

Object	R (Ω)
100 W light bulb	144
60 W light bulb	240
dry human	100,000
wet human	1,000
1 m copper wire	0.02
AA battery	0.10

The image above is an example of a typical resistor found in electronic circuits. The color bands indicate the resistance.

Simple Circuit and Ohm's Law





The circuit diagrammed above is called a "simple" circuit; it consists of one battery, one resistor, and connecting wires.

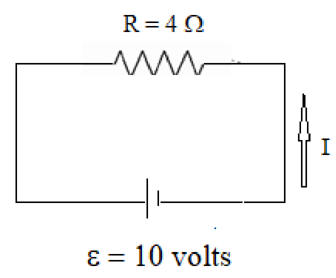
Note above the symbols used to represent a battery and a resistor.

Ohm's Law:

$$I = \varepsilon/R$$

Ohm's Law can be applied only to "simple" circuits.

Example:



(a) What is the current in the circuit above?

$$
\begin{aligned}
I &= \varepsilon/R \\
&= 10/4 \\
&= 2.5\text{ A}
\end{aligned}
$$

(b) What is the power output of the battery?

$$
\begin{aligned}
P &= I\varepsilon \\
&= (10)\,(2.5) \\
&= 25\text{ W}
\end{aligned}
$$

Alternative Battery Power Equation

The "PIE" equation below can be used to determine the power output from any battery in any circuit, whether it's a simple circuit, or not.

$$P = I\varepsilon$$

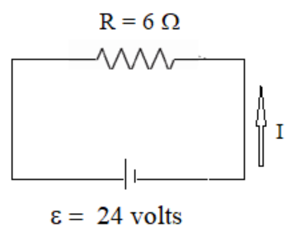
If a circuit is simple, then Ohm's Law may be used to replace I above with ε/R:

$$
\begin{aligned}
P &= (\varepsilon/R)\,\varepsilon \\
&= \varepsilon^2/R
\end{aligned}
$$

Note carefully: Unlike the PIE equation, which can be used for *any* battery in any circuit, this equation is valid only if the circuit is *simple*…only one battery, one resistor.

<u>Example</u>:

In the simple circuit below, a 6 Ω resistor (light bulb) is connected to the terminals of a 24-volt battery. What is the power output of the battery?



$$P = 24^2/6$$
$$= 96 \text{ W}$$

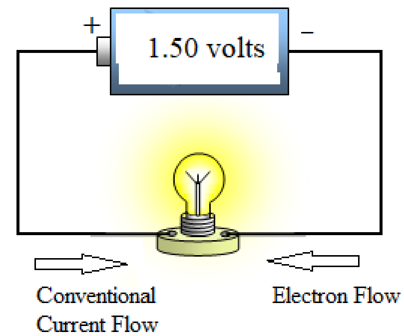
Alternatively, we could use Ohm's Law to find the current:

$$I = 24/6$$
$$= 4 \text{ A}$$

Then use $P = I\varepsilon$
$$= 4(24)$$
$$= 96 \text{ W}$$

Conventional Current

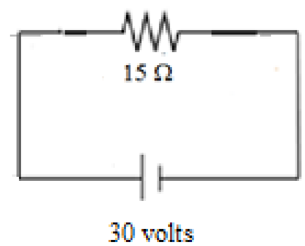
The circuit below shows electrons leaving the negative terminal of the battery and traveling clockwise around the circuit. This is a correct representation of the movement of charged objects in this circuit. Also shown is the direction of flow of imaginary *positive* charges that make up what is called "conventional current."



In previous circuit diagrams in this chapter, the current flow arrows pointed in the actual direction along which charge flowed, i.e., the direction along which *electrons* traveled. From now on in this chapter, only *conventional* current arrows will be shown.

<u>Example</u>:

Determine (a) the conventional current and its direction in the resistor below, and (b) the power output, and (c) the power consumed by the resistor.



(a) $I = \varepsilon/R$
$= 30/15$
$= 2$ A clockwise

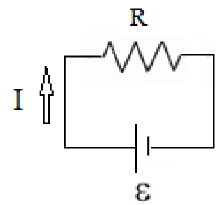
(b) $P = I\varepsilon$
$= (2)(30)$
$= 60$ watts

This is a simple circuit, so the alternative battery power output equation may also be used:

$P = \varepsilon^2/R$
$= 30^2/15$
$= 60$ watts

(c) By the conservation of energy principle we discussed in a previous example, energy produced is energy that is consumed. Therefore, the power consumed is 60 watts.

Power Consumed by a Resistor



The power consumed by the resistor equals the power produced by the battery:

$$P = \varepsilon^2/R \qquad \text{Equation 1}$$

First, note that by Ohm's Law,

$$\varepsilon = IR \qquad \text{Equation 2}$$

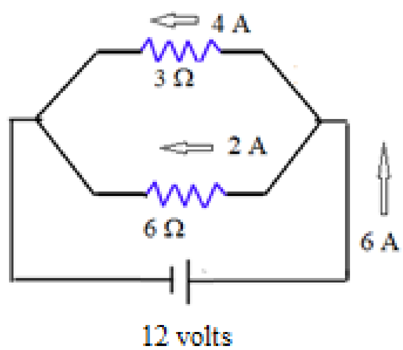
Substitute: $P = (IR)^2/R$

$= I^2R$

$$P = I^2R$$

Example:

(a) Find the powers consumed in the resistors in the circuit below.
(b) What is the total power consumed?
(c) What is the power produced?



(a) 3 Ω: $P = I^2R$
$= 4^2\,(3)$
$= 48$ watts

6 Ω: $P = I^2R$
$= 2^2\,(6)$
$= 24$ watts

(b) Sum of powers consumed: 72 watts

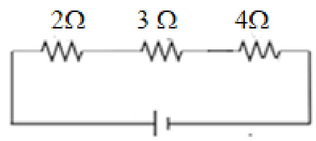
(c) $P = I\varepsilon$
$= (6)(12)$
$= 72$ watts (power produced)

Note that the sum of the powers consumed equals the power produced.

Multi-Resistor Circuits

Series Resistors

Two or more resistors are connected "in series" if the current that goes through one resistor goes through *all* of them. The three resistors below are in series:

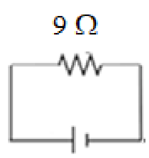


27 volts

The current through the first resistor has no alternative but to pass through the second one, and then through the third one.

Resistors connected in series look the same to a battery as does an imagined *single* resistor--called the "equivalent resistor"--whose resistance is the sum of the resistances.

The simple circuit below is equivalent to the one above, as far as the battery is concerned. The current from the battery is the same in the actual circuit as it is in the imagined equivalent circuit:



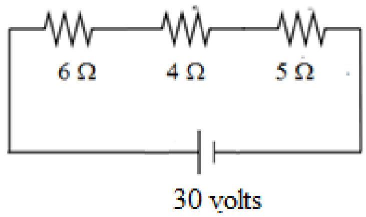
27 V

Summary: For series-connected resistors, the equivalent resistance is found using the "series resistance equation":

$$R = R_1 + R_2 + R_3$$

Example:

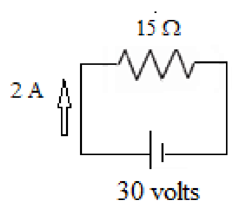
What is the power output of the battery in the circuit below?



The equivalent resistance is

$$R = 6 + 4 + 5$$
$$= 15\ \Omega$$

The equivalent simple circuit is shown below. Using Ohm's Law, we calculate and include the current in the diagram:



$$I = \varepsilon/R$$
$$= 30/15$$
$$= 2\ A$$

$$P = I\varepsilon$$
$$= 2\ (30)$$
$$= 60 \text{ watts}$$

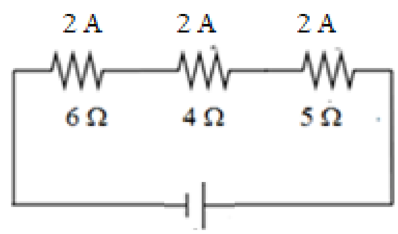
or

$$P = \varepsilon^2/R$$
$$= 30^2/15$$
$$= 60 \text{ watts}$$

Example A:

The example on the previous page showed that the current from the battery was 2.0 A. The resistors are connected in series, so the same 2.0 ampere current flows through each resistor.

Calculate the power consumed in each of the three resistors, and confirm that the sum of the consumed powers equals the produced power of 60 W calculated for the same circuit in the previous example.



Use $P = I^2R$:

5 Ω: $(2)^2\ 5 = 20$ W
4 Ω: $(2)^2\ 4 = 16$ W
6 Ω: $(2)^2\ 6 = 24$ W

Sum of powers consumed = 20 + 16 + 24
= 60 watts

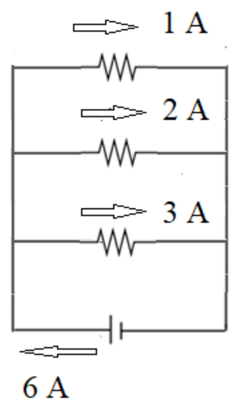
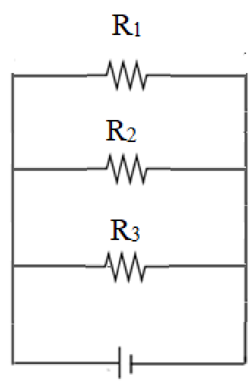
Example B:

The power output of a battery is 100 W. One of the two consumers in the circuit consumes 65 W of power. What is consumed by the other consumer?

The sum of powers consumed equals the power produced:

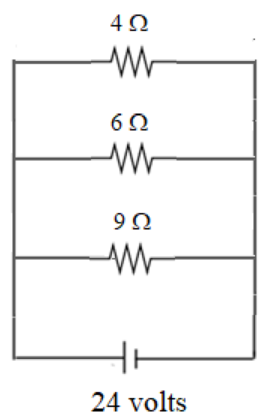
Answer: 100 - 65 = 35 W

Resistors Connected in Parallel

Currents from the battery are distributed unevenly through parallel-connected resistors. The actual distributions depend on the battery voltage and the resistances.	
Resistors connected "in parallel" have an equivalent resistance R given by the "reciprocals equation": $1/R = 1/R_1 + 1/R_2 + 1/R_3$	

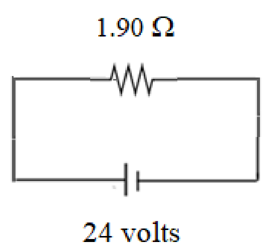
Example:

What is the battery current?



1/R = 1/4 + 1/6 + 1/9
R = 1.90 Ω

The simple circuit below is equivalent to the circuit above:



Ohm's Law applies to simple circuits:

|

I = ε/R
= 24/1.90
= 12.63 A

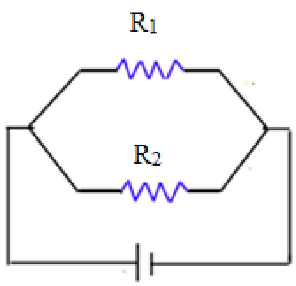
Specialized Parallel Resistance Equation

Two resistors connected in parallel occur so often that it will be convenient to obtain the expression for the equivalent resistance.

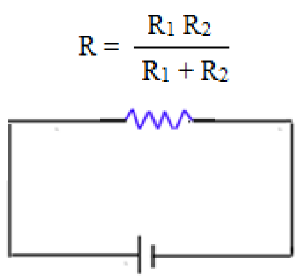
$$1/R = 1/R_1 + 1/R_2$$
$$= R_2/(R_1R_2) + R_1/(R_1R_2)$$
$$= (R_2 + R_1)/(R_1R_2)$$

Reciprocate both sides:

$$R = \frac{R_1 R_2}{R_1 + R_2}$$



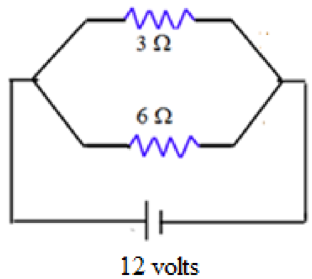
Actual Circuit



Equivalent Circuit

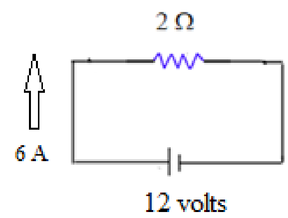
Example:

What is the power output of the battery below?



$R = R_1R_2 / (R_1 + R_2)$
$= (3)(6)/(3 + 6)$
$= 2\ \Omega$

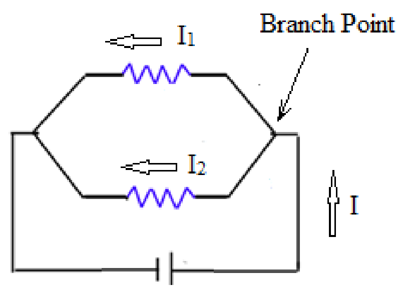
Equivalent Circuit:



$P = I\varepsilon$
$= 6\ (12)$
$= 72$ watts
or
$P = \varepsilon^2/2$
$= 12^2/2$
$= 72$ watts

Calculating Branch Point Currents

The point in a circuit where the current divides or combines is called a "branch point." In the circuit below, part of the current flows into the upper "branch," and the rest into the lower branch.



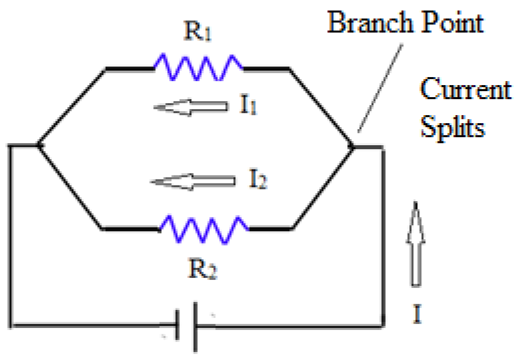
How does one determine the branch currents I_1 and I_2?

The answer is below.

The “Other Divided by the Sum Rule”

(Valid only for <u>two</u> resistors in parallel.)

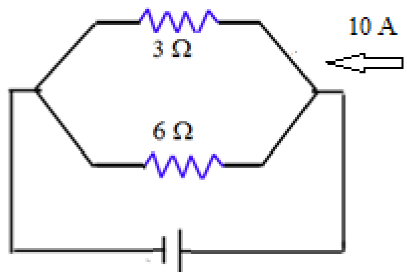
The figures below illustrate the rule.



$$I_1 = \overbrace{R_2}^{\text{The "other one."}} / \underbrace{(R_1 + R_2)}_{\text{the sum}}\, I \qquad I_2 = \overbrace{R_1}^{\text{The "other one."}} / \underbrace{(R_1 + R_2)}_{\text{the sum}}\, I$$

Example:

Calculate the currents in the upper and lower branches of the circuit below.



3 Ω Upper Branch:

Other Resistance = 6 Ω
Sum = 9 Ω

I = (6/9)(10)
= 6.67 A

6 Ω Lower Branch:

Other Resistance = 3 Ω
Sum = 9 Ω

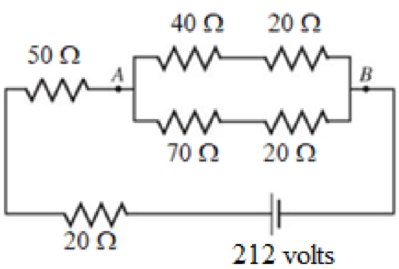
I = (3/9)(10)
= 3.33 A

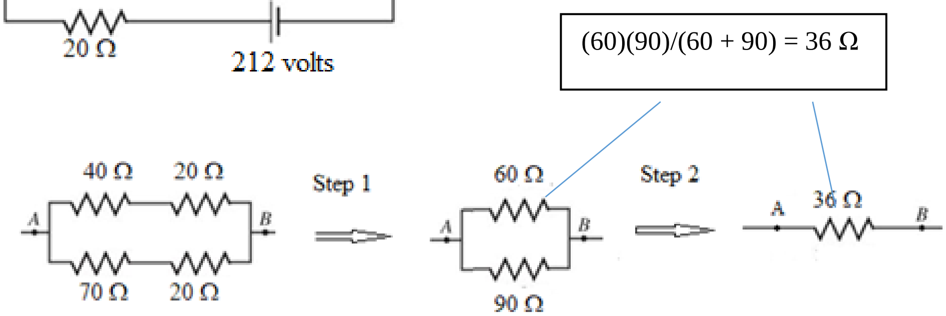
Note that the sum of the two branch currents is 10.00 A, as expected.

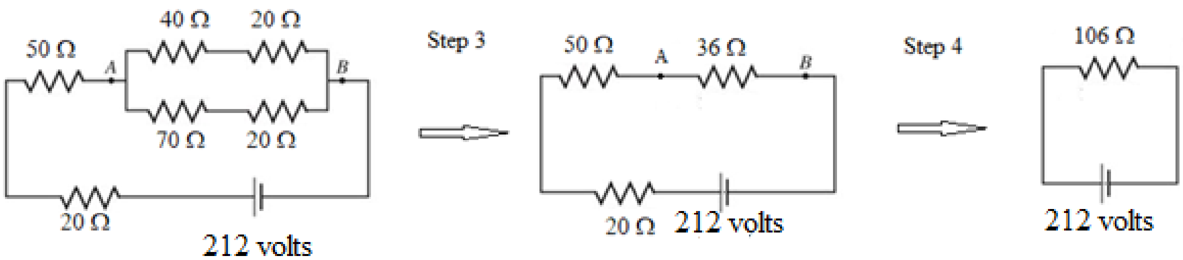
Series and Parallel Resistor Circuits

Example:

What is the power output of the battery below?







$$\begin{aligned} P &= \varepsilon^2/R \\ &= 212^2/106 \\ &= 424 \text{ watts} \end{aligned}$$

Resistor Voltage

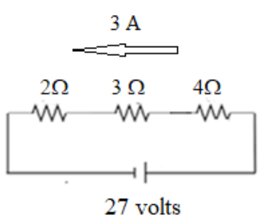
Until now, the only "voltage" we've discussed is the EMF of a battery, which represents the energy provided by the battery to each coulomb of charge leaving the battery.

There is a second kind of "voltage," and it's associated with a current-carrying resistor. This type of voltage represents the energy *consumed* from each coulomb of charge that passes through the resistor.

The name of this other kind of voltage is given below:

"Resistor Voltage"
Units: volts
Symbol: V
V = IR

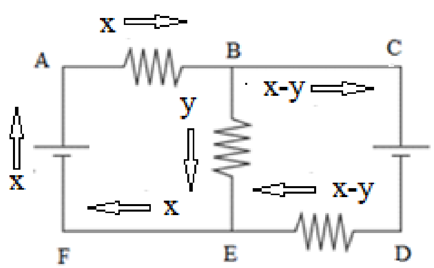
For example, the three resistor voltages in the circuit below are, from left to right, 6 volts, 9 volts, and 12 volts.



We will put the concept of resistor voltages to use when we next analyze circuits more complex than the ones we have discussed so far.

Assigning Current Symbols and Directions

The first step in analyzing complex circuits is to arbitrarily assign direction and symbols for the currents in the various parts of the circuit. That process is illustrated below:



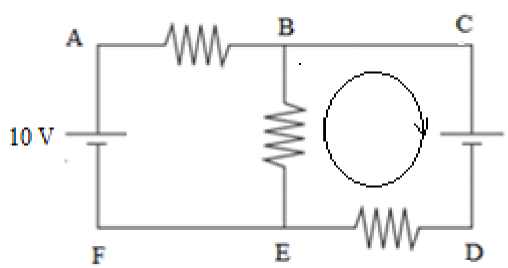
Note that at each branch point the currents must "balance." For example, at branch point B the sum of the currents leaving B equals the current approaching B:

$$y + (x - y) = x$$

Note also that at branch point E the two currents approaching E combine to equal x leaving:

$$(x - y) + y = x$$

Circuit Loops



A "loop" is any closed path around a circuit or portion of a circuit. The BCDEB loop is shown above. Other loops are indicated below:

DEBCD (same as BCDEB)
ABEFA
DEFABCD

We soon will show how loops are used to analyze complex circuits.

Signed Voltages in Loops

Let an imaginary "observer" travel around a loop: It passes through batteries and resistors, and encounters what are called, "signed" voltages. The values and signs of these voltages are determined as explained below:

Signed Voltage of a Battery

A signed voltage of a battery is the battery's voltage preceded by a positive or negative sign.

$$\text{Signed Battery Voltage} = \pm\, \varepsilon$$

Use the positive sign if the direction of travel is from the negative battery terminal toward the positive terminal. Use the negative sign if the direction of travel is from the positive terminal toward the negative terminal.

Signed Resistor Voltages

When one travels through a current-carrying resistor, one encounters the resistor's signed voltage, calculated as described below. Recall that resistor voltages are $V = IR$, but note that *signed* resistor voltages are not the same thing as resistor voltages. Signed resistor voltages can be positive, or negative, while resistor voltages are always positive.

$$\text{Signed Resistor Voltage} = \pm\, IR$$

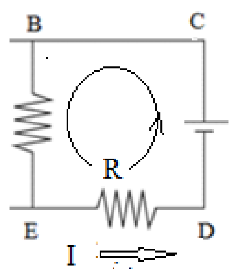
Illustrations of signed resistor voltages are given below:

Signed Resistor Voltages Rules

A signed resistor voltage is +IR if the observer is traveling through the resistor *against* the current arrow; it is negative if the observer is traveling *with* the current arrow.

Suppose the current through the resistor below is arbitrarily guessed to be to the right. Suppose further we choose to travel counter-clockwise around the loop EBCDE. The travel direction through the resistor is therefore *with* the current arrow, so, by the rule above, we precede the IR quantity with a negative sign.

$$V = - IR$$

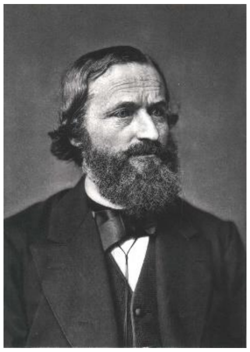


If we were to imagine that our observer were traveling clockwise, the travel direction through the resistor would be *against* the current arrow, so, by the rule, the signed resistor voltage is positive:

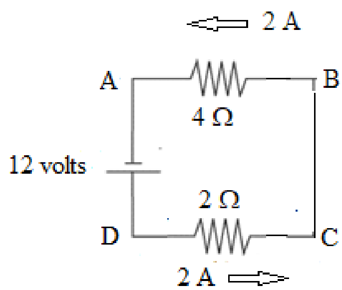
$$V = IR$$

Kirchoff's Law

"The sum of the signed voltages around a circuit loop is zero."--Gustav Kirchoff



Consider the circuit below:



Traveling around loop clockwise ABCDA, the sum of the signed voltages is zero:

2 (4) + 2 (2) - 12 = 0

If we were to travel counter-clockwise ADCBA, the sum is still zero:

12 - 4 - 8 = 0

Kirchoff's Law is sometimes called "The Loop Rule."

Use Kirchoff's Law to Solve Complex Circuits

The first step in analyzing a complex circuit is to <u>guess</u> the various directions of the current arrows entering and leaving the branch points. Make sure that the sum of the currents entering and leaving a branch point "balance," i.e., the current in equals the current out. Examples current balance are in Figures 1 and 2, below

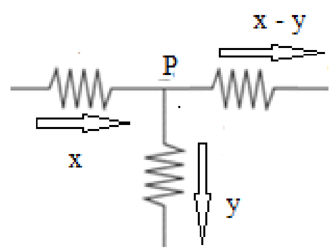


Figure 1

Sum in: x
Sum out: $(x - y) + y = x$

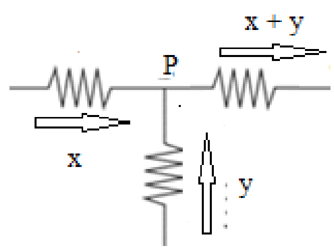
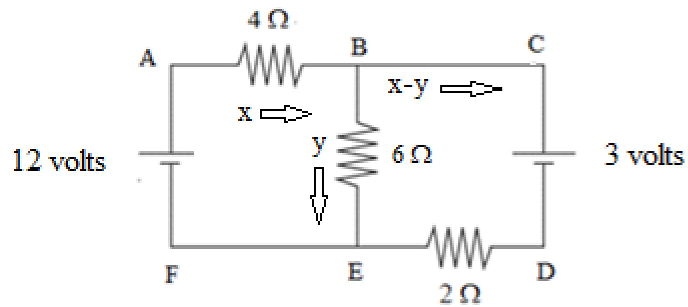


Figure 2

Sum in: $x + y$
Sum out: $(x + y)$

Example: Determine the currents x, y in the circuit below.

Let the current through the 4-ohm resistor below be labeled "x," with the guessed direction of current as shown. Similarly, the guessed direction of current y is indicated.



Apply Kirchoff's Law to the two arbitrarily chosen loops below:

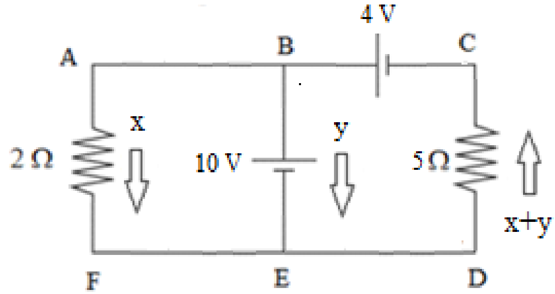
ABEFA: $-4x - 6y + 12 = 0$
BCDEB: $-3 - 2(x - y) + 6y = 0$

Solve this pair of equations for x and y:

$x = 1.77$ A
$y = 0.82$ A

Example:

Apply Kirchoff's Law to find the currents x and y.



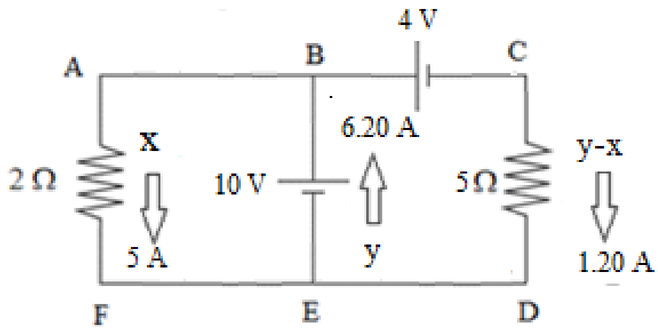
ABEFA:
$-10 + 2x = 0$
x = 5.00 A

BCDEB:
$-4 + 5x + 5y + 10 = 0$
y = -6.20 A

Note: If one guesses wrong about the direction of a current, the current will be negative. In such a case, <u>remove the negative sign and reverse the direction of the current arrow</u> obtain the correct current and direction.

In the present case, the current y is negative, so we remove the negative signs and reverse the arrow:

The correct circuit currents are shown below.



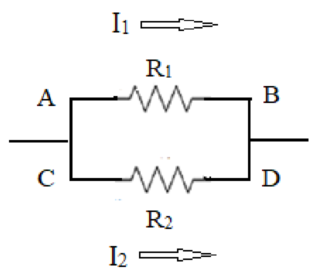
Note: the 1.20 A current is obtained by balancing the currents at branch point B. No longer is the current from C to D symbolized as x + y. Now, it's y -x.

The sum of the currents leaving B equals the current entering:

Entering: 6.20 A
Leaving: 5.00 A + 1.20 A

Parallel Resistor Voltages

Use the loop rule to prove that the resistor voltages IR of parallel resistors are the same.



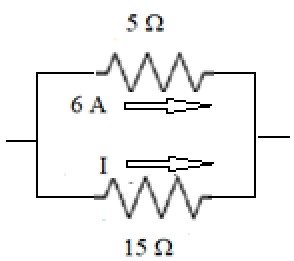
ABDCA: $-I_1 R_1 + I_2 R_2 = 0$

$$I_1 R_1 = I_2 R_2$$

$$V_1 = V_2$$

Example:

Using the fact that parallel resistors have the same resistor voltage, determine the current I in the circuit below.

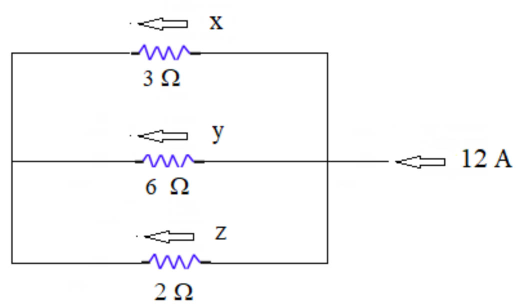


$I\,(15) = (6)(5)$

$I = 2$ A

Example:

Use the parallel-resistor voltages rule to find the currents in the circuit below. We need three equations to solve for the three unknowns.



First Equation

Equating the top two IR's:

$3\,x = 6\,y$

$x = 2y$

Second Equation

Equate the bottom two IR's:

$6\,y = 2z$

$z = 3\,y$

Third Equation

Sum of currents = 12 A

$x + y + z = 12$

$(2\,y) + (y) + (3y) = 12$

y = 2 A

....

$x = 2y$

x = 4 A

$z = 3y$

z = 6 A

Summary of Important Circuit Quantities and Equations

Symbol	Name	Units
R	Resistance	Ohms (Ω)
Q	Quantity of Charge	Coulombs (C)
E	Battery Energy Given to Exiting Charge	Joules (J)
ε	Electromotive Force (EMF)	Volts (V)
I	Current	Amperes (A)
P	Power	Watts (W)
$\varepsilon = E/Q$	Battery EMF (Voltage)	Volts (V)
$I = Q/t$	Current	Amperes (A)
$I = \varepsilon/R$	Battery Current (simple circuit only)	Amperes (A)
$P = I\varepsilon$	Battery Power Output	Watts (W)
$P = \varepsilon^2/R$	Power Output (simple circuit only)	Watts (W)
$P = I^2R$	Power Consumed by a Resistor	Watts (W)
$V = IR$	Resistor Voltage	Volts (V)

Summary of Important Equations

$\varepsilon = E/Q$	Energy per Coulomb
$I = Q/t$	Coulombs per Second
$I = \varepsilon/R$	Ohm's Law (Simple Circuit
$P = I\varepsilon$	Output Power
$P = \varepsilon^2/R$	Output Power (Simple Circuit)
$P = I^2R$	Power Consumed by a Resistor
$V = \pm IR$	Signed Resistor Voltage
$V = \pm\varepsilon$	Signed Battery Voltage
$R = R_1 + R_2$	Equivalent Resistance (Series)
$R = R_1R_2/(R_1+R_2)$	Equivalent Resistance (Parallel)
$I_1 = R_2/(R_1+R_2)\ I$	Upper Branch Current
$I_2 = R_1/\ (R_1+R_2)\ I$	Lower Branch Current