Physics 25 Chapters 16-17

Waves

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Video Lecture 1: String Wave Resonances
Video Lecture 3: Doppler Effect for Sound
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Video Lecture 5: Open-Open Tube Resonances
Video Lecture 6: Sound Intensity, Decibels
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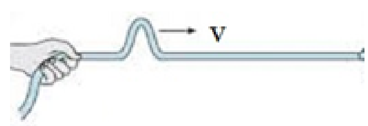
Waves on Strings



Oscillate (shake) once: A "pulse" is created that travels to the right at a pulse speed v

Waves

Wave Frequency

Oscillating the string repeatedly creates a “train” of pulses, called a “wave,” whose “frequency” is the number of times per second the string is oscillated (shaken).

Equivalent SI units of frequency units are shown below:

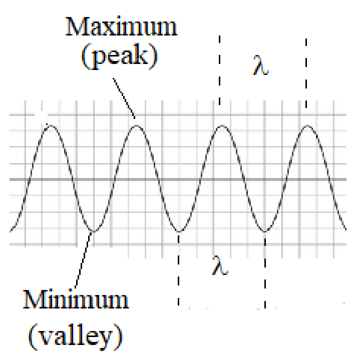
Units
s^{-1} (per second)
hertz (Hz)

For example, a frequency of five oscillations per second may be indicated in either one of two equivalent units:

$5.0\ s^{-1}$
5.0 Hz

Wavelength

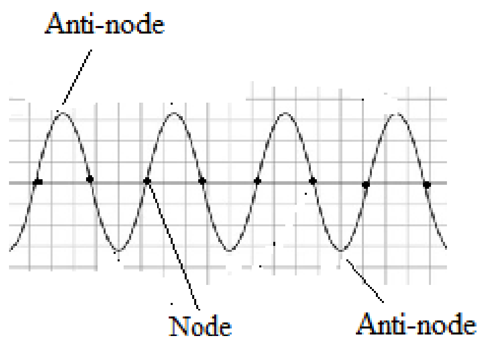
At any instant, various points on the string are displaced vertically by various amounts. Points of extreme displacements occur at “peaks” (maxima) and “valleys” (minima). The distance between two maxima is the same as the distance between two minima, and is called the “wavelength” of the wave, symbolized as λ.



Nodes and Anti-nodes

As the wave travels to the right there are places where the string is momentarily not displaced, as shown below. These places are called “nodes.”

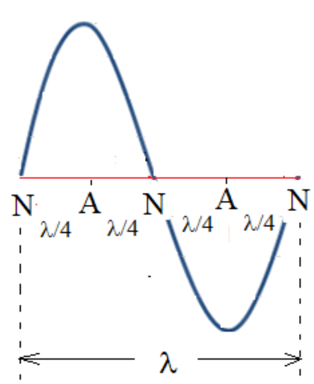
Maxima and minima are places along the length of the string which are experiencing momentary extremes, i.e., either maxima, and minima. These occur between two nodes, and are called “antinodes.”



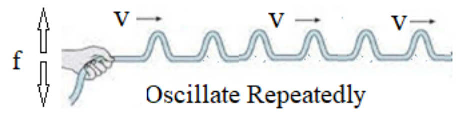
Note below that the distance between two nodes is one-half of a wavelength ($\lambda/2$. Midway between two nodes is an antinode, so we can further note that the distance between a node and an antinode is one-fourth of a wavelength, $\lambda/4$:

$$\overline{NA} = \lambda/4$$

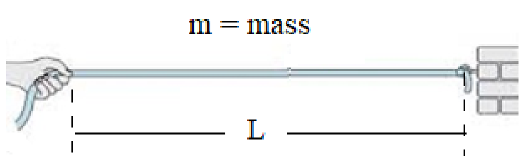
$$\overline{NN} = \lambda/2$$



Wave Speed



Each of the pulses that make up the wave is traveling at the same speed, v, called the "wave speed." The wave speed depends on the mass m and length L of the string, and on the string's tension T.



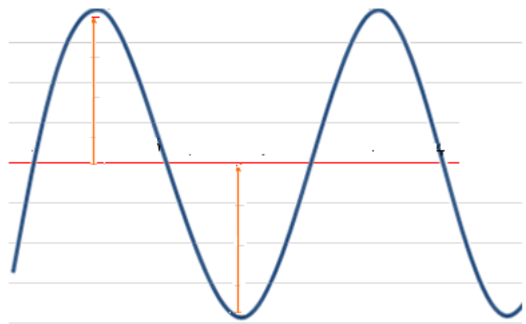
μ = "Linear Mass Density"
= m/L
T = Tension
v = Pulse Speed
= $(T/\mu)^{1/2}$

<u>Example</u>:

T = 5.0 N
m = 0.04 kg
L = 0.80 m
μ = 0.04 kg/(0.80 m_
= 0.05 kg/m
v = $(5.0/0.05)^{1/2}$
= 10 m/s

Amplitude

The "amplitude" of a string wave is the absolute value of the maximum displacement of the string particles. Amplitudes will not play a role in the coming discussion of waves, so we don't bother to provide amplitude a symbol name.



The Wave Equation

The three quantities wavelength λ, wave frequency f, and wave speed v are related to each other through the so-called "wave equation, " shown below:

$$\lambda f = v$$

Example A:

The speed of waves on a string is 12.0 m/s.

At what frequency will the wavelength of wave on the string be 4.0 m?

$f = v/\lambda$
$= (12.0 \text{ m/s})/(4.0 \text{ m})$
$= 3.0 \text{ s}^{-1}$
$= 3.0 \text{ Hz}$

Example B:

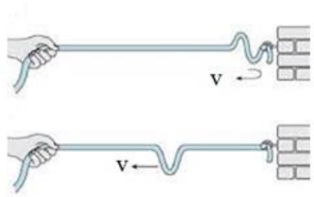
Calculate the wavelength of the string wave that has the following attributes:

T = 100 N	m = 0.40 kg	L = 1.6 m	f = 10 Hz

$\mu = (0.40)/1.6$
$= 0.25$
$v^2 = 100 / 0.25$
$v = 400^{1/2}$
$= 20 \text{ m/s}$

$\lambda = (20 \text{ m/s})/10 \text{ s}^{-1}$
$= 2 \text{ m}$

String Wave Resonances



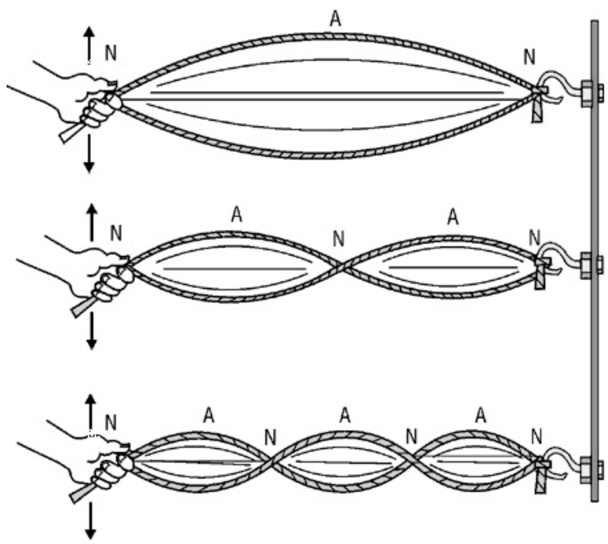
If the string is tied to a wall at the right end, pulses reflected from the wall bounce back and forth from the wall to the hand, hand to wall.

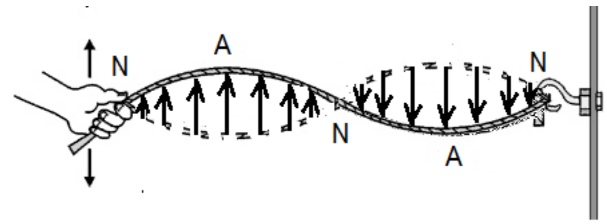
At certain frequencies, the oppositely-traveling pulses overlap "constructively," meaning that a wave is created whose wave amplitude is greater than the oscillation amplitude. This effect is called a "resonance."

Another name for resonance is "standing wave" because the wave's shape, to the eye, appears to be unchanging.

Examples of standing waves are shown below.

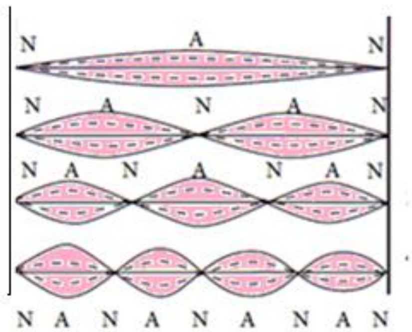
The resonances below are in the shape of a series of "loops." At the ends of each loop are nodes. Recall that the distance between consecutive nodes in a wave is λ/2. Thus, each loop has a width of half of a wave-length (λ/2).





String particles in adjacent loops are oscillating "out of phase": If the string in one loop is moving upward at some moment, the string in any adjacent loop is moving downward, and vice-versa.

Click Here to view video demonstration of standing waves.



The standing waves above are in the shape of a series of “loops.” At the ends of each loop are nodes. The A and N labels alternate. The reader is to understand that the right ends of the strings shown above are tied down, while the left ends are attached to an oscillator whose vibration amplitudes are small enough compared to the ones at the antinode locations that we can treat those ends as nodes.

Recall that the distance between consecutive nodes in a wave is a half-wavelength, $\lambda/2$. Thus, each loop above has a width equal to a half wave-length: $\lambda/2$.

The figures above show standing waves with one, two, three, and four loops. Because loops have one antinode at their centers, we may alternatively say that the figures above show one, two, three, and four-*antinode* standing waves.

Example:

We show below that the frequencies that resonate on a string tied at both ends are integer multiples of the lowest resonant frequency:

Shown in the table below are examples of string resonant frequencies and the names associated with them.

The lowest frequency that resonates is called the “fundamental frequency.” Other frequencies are integer multiples (1,2, 3, 4…) times the fundamental frequency. All of these frequencies are called “harmonics.”

Frequencies higher than the fundamental are “over” the fundamental frequency, and are called “overtones.” The next-higher resonant frequency above the fundamental is called the “first” overtone, for example.

Frequency (Hz)	Symbol	Name	Other Name
40	f_1	First Harmonic	Fundamental
80	f_2	Second Harmonic	First Overtone
120	f_3	Third Harmonic	Second Overtone
160	f_4	Fourth Harmonic	Third Overtone

$$f_n = nf_1,\ n = 1, 2, 3, 4, \ldots.$$

Example:

The sixth harmonic frequency resonating on a string is 780 Hz. What is the second harmonic?

$$6\ f_1 = 780\text{ Hz}$$
$$f_1 = 130\text{ H}$$
$$f_2 = 2\ (130)$$
$$= 260\text{ Hz}$$

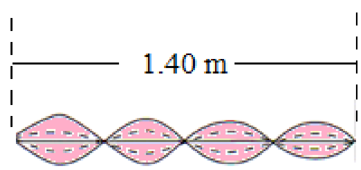
VIDEO DEMONSTRATION OF STANDING WAVE

Click Here

Example:

The speed of pulses on a string of length 1.40 m is 2.60 m/s.

What frequency of oscillation will create a standing wave with four antinodes?



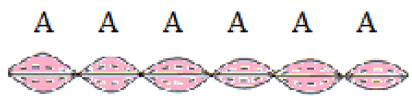
Each of the four loops has a width of $\lambda/2$. The sum of the four loop widths equals the length of the string:*

$$4(\lambda/2) = 1.40\text{ m}$$
$$\lambda = 0.70\text{ m}$$

$$f = v/\lambda$$
$$= 2.60/0.70$$
$$= 3.71\text{ Hz}$$

.

<u>Example</u>:



Resonance on 1.60 m string occurs with six antinodes when the string is oscillated at 6.0 Hz. The tension in the string is 14.0 N.

What is the linear mass density of the string?

Solution:

$(T/\mu)^{1/2} = v$
$\mu = T/v^2$
$= 14.0/v^2$
We need v:

There are six loops, and all loops on strings have a width of λ/2, so

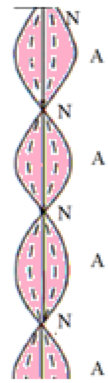
$6(\lambda/2) = 1.60$
$\lambda = 0.53$ m

$v = \lambda f$
$= 0.53\ (6.0)$
$= 3.18$ m/s

$\mu = 14.0/v^2$
$=14.0/3.18^2$
$= 1.38$ kg/m

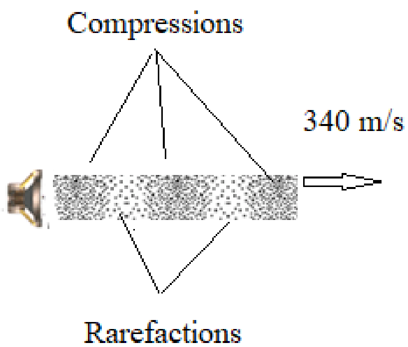
Resonances on Hanging Ropes

If one end of a hanging rope is oscillated at the top, while the bottom is free to move, a standing wave can be made to occur at certain frequencies. When resonance occurs, the top of the rope is a node, while the bottom is an antinode.

<u>Example</u>: A rope 0.90 meters long is hanging vertically; the bottom of the rope is free. An oscillator attached to the top of the rope is vibrating at a frequency of 4.0 Hz, which causes a resonance with four antinodes (3.5 loops). What is the speed of waves on this rope?	 $3.5\ (\lambda/2) = 0.90$ $\lambda = 0.51$ m $v = \lambda f$ $= 0.51\ (4.0)$ $= 2.1$ m/s

Sound Waves

The figure below shows a speaker whose vibrating membrane is creating a sound wave consisting of a train of compressions and "rarefactions" moving at the "speed of sound," 340 m/s. Compressions are, as the name implies, places where the air is more dense than normal, while rarefactions are places where the air is less dense.



The Figure 1 below indicates that the wavelength λ of a sound wave is the distance between neighboring compressions. Figure 2 shows that the wavelength is also the distance between neighboring rarefactions.

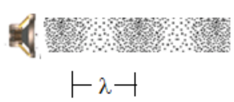


Figure 1

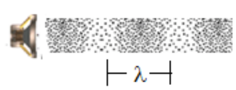


Figure 2

The Wave Equation for Sound

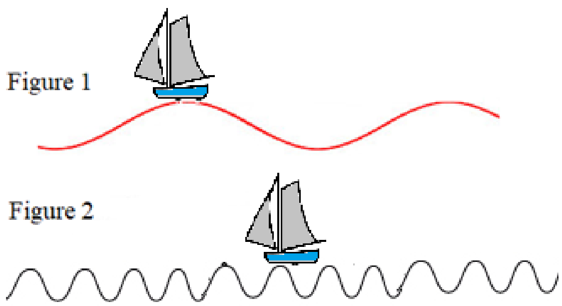
$$\lambda f = 340 \text{ m/s}$$

f = Frequency of the Sound Source

Waves Interfering with Matter

Waves whose wavelengths are much longer than the width of an object do little or no damage to the object, while waves with wavelengths comparable to the size of an object can exert shearing forces that could rip he object apart.

Consider the example below of a ship at sea. In Figure 1 the ship's rises and falls are gentle, while the experience of the ship in Figure 2 is like that which would be experienced by an automobile traveling at high speed over a succession of closely spaced speed bumps.



In Figure 1, the ship experiences gentle rises and falls, while the ship in Figure 2 experiences sudden violent rises and falls which could tear the ship apart.

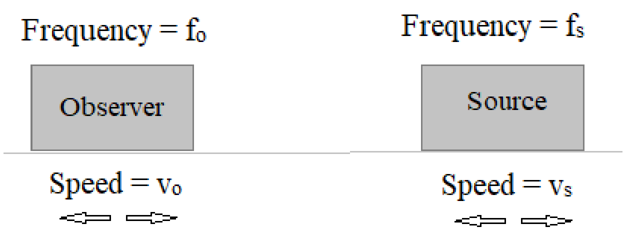
<u>Example</u>:

"Lithotripsy" is a medical procedure in which ultrasound is used to break up kidney stones. What is the optimum ultrasound frequency for breaking up a kidney stone whose diameter is one centimeter (0.01 m)?

In kidney tissue, the speed of sound is about 1500 m/s.

$f = v/\lambda$
$= 1500/0.01$
$= 150{,}000$ Hz

The Doppler Effect for Sound



$$f_o = f_s \frac{(340 \pm v_o)}{(340 \pm v_s)}$$

Rule: If the object (source, or observer) is moving away from the other one, use the sign for that speed term that makes the ratio smaller. Mnemonic: "away" and "smaller" have negative connotations.

If it's moving toward the other one, use the sign for that speed term that makes the ratio larger. Mnemonic: "toward" and "larger" have positive connotations.

See Video Explaining Doppler Effect for Sound

Example A:

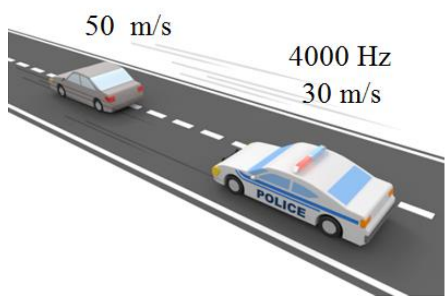
Suppose an observer is at rest ($v_o = 0$), and a fire truck with its siren on is moving away from the observer at a speed $v_s = 30$ m/s. If the siren's frequency f_s is 5,000 Hz, what does the observer hear?

The source is moving away, so, by the rule given above, we need to choose the sign for the source speed that will make the ratio smaller, so we make the denominator *larger* by *adding* 30 to the 340 in the denominator.

$f_o = 5000\ (340 + 0) / (340 + 30)$
$= 4595$ Hz

Example B:

A police car emitting 4000 Hz sound is chasing a speeder. What frequency does the driver of the speeding car hear?

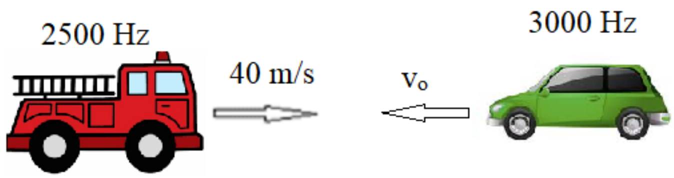


$f_o = 4000\ (340 - 50) / (340 - 30)$
$= 3742$ Hz

Example A:

A fire truck emitting 2500 Hz and traveling at 40 m/s is racing toward an automobile that's traveling toward the fire truck. The automobile driver hears 3000 Hz.

What is the automobile's speed?



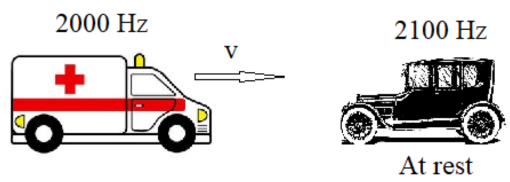
Each vehicle is moving toward the other, so each sign choice is the one that makes the ratio larger: We make the numerator larger by adding, and the denominator smaller by subtracting.

$3000 = 2500\ (340 + v_o)\ /\ (340 - 40)$

$v_o = 20$ m/s

Example B:

An ambulance emitting 2000 Hz is moving toward a stationary auto; the auto's occupant hears 2100 Hz. What is the ambulance's speed?



The ambulance is moving toward the observer, so we choose the sign for the source speed that makes the ratio larger: the negative sign.

$2100 = 2000\ (340 + 0)/(340 - v_s)$

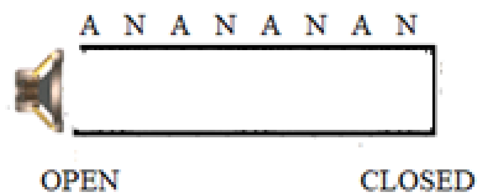
$v_s = 16.2$ m/s

Sound Wave Resonance in Hollow Tubes

Shown below is a hollow tube, closed at the right end, and open at the left end. We will refer to such a tube as "open-closed."

A vibrating speaker membrane causes air in the tube to vibrate back and forth. At certain frequencies a "resonance" occurs which results in much louder sound than would otherwise be heard if the sound were not being directed into the tube.

When sound is resonating in the tube, there is a displacement anti-node at the open end, and a displacement node at the closed end. Resonances can occur with more than just one node and one antinode; shown below is a resonance with four nodes and four antinodes. At any of the nodes, air is neither moving rightward, nor leftward; it is stationary. At nodes, the air is not moving, and has the density of undisturbed air (the air in the room, for example). On the other hand, the air at any anti-node is oscillating leftward and rightward, becoming alternately condensations, then rarefactions. At nodes, the air is not moving, and has the density of undisturbed air.



When resonance occurs there is always a displacement node at the closed end of the tube, analogous to the displacement node that always exists at the end of the string attached to the clamp, as well as at the end at which the hand or other agent is vibrating the string. At nodes in resonating sound in tubes, the air is not moving, analogous to the motionless string particles in resonating strings.

Just like the antinode at the bottom of a hanging rope that's experiencing resonance, the open end of the tube is an antinode: air particles at antinodes experience maximum displacement back and forth, to the right, and then to the left.

Note: Just as was true about displacement nodes and antinodes in string wave resonances, the distance between a neighboring sound wave displacement nodes and antinodes is a quarter wavelength:

$$\overline{AN} = \lambda/4$$

When sound wave resonance occurs in tubes, the following is true:

- Closed ends are displacement nodes.
- Open ends are displacement antinodes.
- A and N labels alternate:

$$AN = NA = \lambda/4$$

When this happens, small-amplitude sound input at an open end of a tube becomes amplified.

Harmonic Frequencies

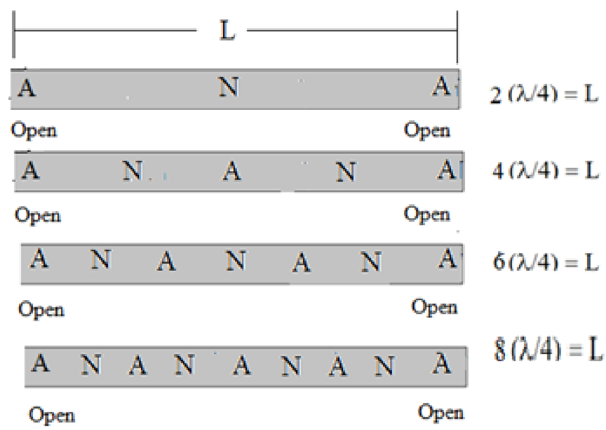
The frequencies that resonate in hollow tubes are often called "harmonic" frequencies. Often, the words "harmonic frequencies" is shortened to "harmonics." We will show in the work below how these frequencies are calculated for open-open tubes first, and then later we will do the same thing for open-closed tubes.

Open-Open Tubes

Example:

What are the four lowest harmonics that resonate in tubes open at each end, and 0.85-meter tube long?

The sum of the quarter-wavelengths ($\lambda/4$) distances AN and NA equals the length of the tube.



$2(\lambda/4) = 0.85$ m $\lambda = (3.40/2)$ m $f_1 = 340/\lambda$ $= 200$ Hz Fundamental or First Harmonic	$4(\lambda/4) = 0.85$ m $\lambda = (3.40/4)$ m $f_2 = 340/\lambda$ $= 400$ Hz Second Harmonic	$6(\lambda/4) = 0.85$ m $\lambda = (3.40/6)$ m $f_3 = 340/\lambda$ $= 600$ Hz Third Harmonic	$8(\lambda/4) = 0.85$ m $\lambda = (3.40/8)$ m $f_4 = 340/\lambda$ $= 800$ Hz Fourth Harmonic

Harmonics are integer multiples of the fundamental frequency:

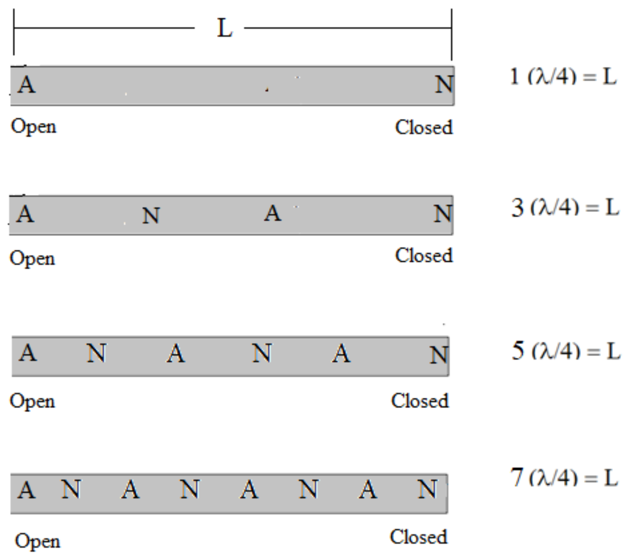
$f_n = nf_1$, n = 1, 2, 3, 4….

Resonance in Open-Closed Tubes

<u>Example</u>:

What are the four lowest harmonics of sound in open-closed tube 0.85 meter long? By the rules listed above, there is a node (N) at the closed end, and an antinode (A) at the open end. Further note that the letters A and N alternate, as was the case for string waves, and open-open tubes.

The sum of the quarter-wavelengths ($\lambda/4$) distances AN and NA equals the length of the tube.



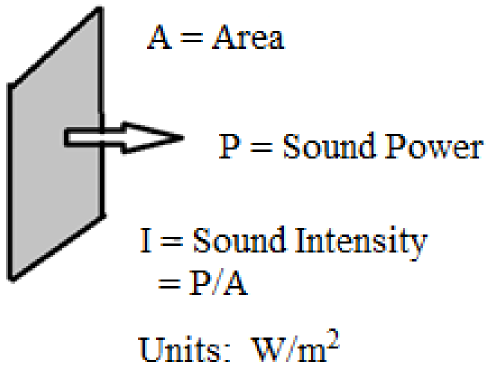
$1(\lambda/4) = 0.85$ $\lambda = 3.40$ m $f = 340/\lambda$ $= 100$ Hz $f_1 = 100$ Hz 1st Harmonic	$3(\lambda/4) = 0.85$ $\lambda = (3.40/3)$ m $f = 340/\lambda$ $= 300$ Hz $f_3 = 300$ Hz 3rd Harmonic	$5(\lambda/4) = 0.85$ $\lambda = (3.40/5)$ m $f = 340/\lambda$ $= 500$ Hz $f_5 = 500$ Hz 5th Harmonic	$7(\lambda/4) = 0.85$ $\lambda = (3.40/7)$ m $f = 340/\lambda$ $= 700$ Hz $f_7 = 700$ Hz 7th Harmonic

The harmonics in open-closed tubes are *odd*-integer multiples of the fundamental frequency:

$$f_n = nf_1, n = 1, 3, 5, 7, ..$$

For example, if the lowest harmonic is 40 Hz, then the other harmonics are 120 Hz, 200 Hz, 280 Hz….

Sound Intensity



Example:

Each second, 4.0×10^{-6} J of sound energy land on an area $A = 2.0 \times 10^{-4}\ m^2$.

What is the sound intensity at that location?

$P = 4.0 \times 10^{-6}$ W
$I = P/A$
$= (4.0 \times 10^{-6}\ W)/(2.0 \times 10^{-4}\ m^2)$
$= 2.0 \times 10^{-2}\ W/m^2$

Spherically-Symmetrical Sound Sources

Sound sources that broadcast equal amounts of energy each second in all directions are called “spherically-symmetric” sound sources. If the power output of the source of sound is P, and the listener’s ear is a distance r away, located on the surface of an imaginary sphere whose surface area is $4\pi r^2$, the ear of the listener experiences a sound intensity according to the equation below:

$$I = P/4\pi r^2$$

Example:

A spherically-symmetric sound source has a power output of 200 watts.

What is the intensity 4.0 meters away?

$A = 4\pi(4.0)^2$
$= 201.06\ m^2$

$I = P/4\pi r^2$
$= (200\ W) / (201.06\ m^2)$
$= 0.995\ W/m^2$

Example A:	Example B:
How far from a spherically-symmetric 40-watt sound source would the sound intensity be one milli-watt per square meter? $I = P/4\pi r^2$ $1.0 \times 10^{-3} = 40 / (4\pi r^2)$ $r = 56.42$ m	At a certain distance from a spherically-symmetric sound source the intensity is 2.0 W/m^2. What is the intensity five times farther away? Quintupling r will cause the denominator to become $5^2 = 25$ times its previous value. Therefore, the new intensity will be 1/25th of the previous value: New intensity = 2.0/25 = 0.08 W/m^2

The Threshold of Human Hearing

The least sound intensity the average healthy human ear can detect is called "The Threshold of Human Hearing."

$$I_o = 1.0 \times 10^{-12} \text{ W/m}^2$$

Below this intensity, the ear hears nothing, hence the subscript zero: I_o

This is the intensity an ear experiences when a nearby butterfly flaps its wings.

Example:

(a) 1.5 meters away from a whisper the sound intensity is 2.0×10^{-11} W/m^2. Treating the whisperer as a spherically-symmetric sound source, what is its output power P?

$I = P/(4\pi r^2)$
$P = 4\pi r^2 I$
$= 4\pi r(1.5)^2\ 2.0 \times 10^{-11}$
$= 5.65 \times 10^{-10}$ W

(b) What is the greatest distance from the whisperer at which the whisper still be heard?

$I = P/(4\pi r^2)$
$1.0 \times 10^{-12} = 5.65 \times 10^{-10}/4\pi r^2$
$r = 6.71$ m

Decibel Level (also called "Sound Level")

$$\beta = 10 \log_{10} (I/I_o)$$

Units: "decibels" (dB)

Example:
$I = 3.0 \times 10^{-6}$ W/m^2
What is the decibel level?
$\beta = 10 \log_{10} (3.0 \times 10^{-6}/1.0 \times 10^{-12})$
$= 65$ dB

Sound Source	I (W/m^2)	β (dB)
Inaudible	1.0×10^{-12}	0
Whisper at one meter	2.0×10^{-11}	13
Bosch Dishwasher at 5 meters	4.0×10^{-8}	46
Normal Conversation at 1.0 meters	3.0×10^{-6}	65
Garbage Disposal at 2 meters	1.0×10^{-5}	70
Front Row Rock Concert (hearing loss)	0.3	115
AR-15 Army Rifle near Ear (rupture eardrum)  (Firing my AR at ear height over a tall hedgerow at Viet Cong in 1970 cost me hearing in my right ear.)	10.0	130

Example:

The decibel level at a certain point is β = 46 dB.

What is the sound intensity I at that point?

$46 = 10 \log (I/I_o)$
$4.6 = \log (I/I_o)$

Use below this relationship: $10^{\log x} = x$

$10^{4.6} = I/I_o$
$I = (1.0 \times 10^{-12}\ W/m^2)\ 10^{4.6}$
$= 3.98 \times 10^{-8}\ W/m^2$

Calculating Changes in Decibel Level

Recall log property: $\log (A/B) = \log A - \log B$

$\beta_2 = 10 \log (I_2/I_o)$
$= 10 \log I_2 - 10 \log I_o$
$\beta_1 = 10 \log (I_1/I_o)$
$= 10 \log I_1 - 10 \log I_o$
$\Delta\beta = \beta_2 - \beta_1$
$= 10 \log I_2 - 10 \log I_1$
$= 10 \log (I_2/I_1)$

Example A: If sound intensity becomes 30 times greater, what would be the change in decibel level? $\Delta\beta = 10 \log (I_2/I_1)$ $= 10 \log (30)$ $= 14.8$ dB

Example B:	Example C:
$I_2 = 2I_1$	$I_2 = ½ I_1$
$\Delta\beta = ?$	$\Delta\beta = ?$
$\Delta\beta = 10 \log(2I_1/I_1)$	$\Delta\beta = 10 \log(½I_1/I_1)$
$= 10 \log(2)$	$= 10 \log (½)$
$= 3.02$	$= -3.02$
$= 3.0$ dB	$= -3.0$ dB
(Approximately)	(Approximately)

The 3-dB Up and 3-dB Down Rules

The examples above show that when the sound intensity is doubled, the decibel level goes up by about 3.0 dB and when the sound intensity is halved, the decibel level goes down by about 3.0 dB. These doubling and halving rules are called the "3-db up," and "3-db down" rules.

Example B: Suppose the sound intensity at a certain point increases to eight times as much a previously. What is the change in decibel level?   Three doublings: 2 x 2 x 2 = 8   Each doubling increments the dB level by about 3 dB: Answer: 3 + 3 + 3 = 9 dB

Example A:

(a) Suppose the intensity at some point is increased to 40 times its previous value. How many doublings (call it N) of intensity is this?

$2^N = 40$
Take the log of both sides, and use the log property: $\log x^y = y \log x$

$N \log 2 = \log 40$
$N = 5.32$

(b) What is the approximate increase in the decibel level? Use the better value of 3.02 dB per doubling.

Answer: 5.32 (3.02) = 16.07 dB

Example B:

Obtain a more precise value of the change in the decibel level for the problem in Example B.

$\Delta\beta = 10 \log(I_2 / I_1)$
$\Delta\beta = 10 \log(40 I_1 / I_1)$
$= 10 \log (40)$
$= 16.02$ dB

Example C:

The decibel level at a certain point is 76 dB. What will be the new decibel level when the sound intensity is increased to nine times its previous value?

$\Delta\beta = 10 \log(I_2 / I_1)$
$\Delta\beta = 10 \log(9 I_1 / I_1)$
$= 10 \log (9)$
$= 9.54$ dB

$\beta = 76 + 9.54$
$= 85.54$ dB

Example:

Twenty-four cages, each containing a barking dog, are arranged in a circle. At the center of the circle is a listener. If she measures a dB level of 96 dB, how many dogs would have to stop barking in order that the sound level drop to 87 dB? Assume the dogs are spherically symmetric sound sources barking with the same power.

96 - 87 = 3 + 3 + 3

Applying the “3 dB down” rule, there will have to be three halvings of sound intensity:

24 halved to 12
12 halved to 6
6 halved to 3
Only three dogs are left barking:
24 - 3 = 21
21 dogs would have to stop barking.